Examiners' Report: Preliminary Examination in Mathematics and Philosophy Trinity Term 2019

November 21, 2019

Part I

A. STATISTICS

(1) Numbers and percentages in each class

See Table 1. Overall, 20 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2019	(2018)	(2017)	(2016)	(2015)	2019	(2018)	(2017)	(2016)	(2015)
Distinction	7	6	4	7	6	35	42.86	23.53	50	42.86
Pass	11	7	13	4	7	55	50	76.47	28.57	50
Partial Pass	2	1	0	3	1	10	7.14	0	21.43	7.14
Fail	0	0	0	0	0	0	0	0	0	0
Total	20	14	17	14	14	100	100	100	100	100

(2) Vivas

No vivas were given.

(3) Marking of Scripts

In Mathematics, all scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. There is an extensive checking process. In Philosophy, all scripts were single marked except for failing scripts, which were double-marked.

B. NEW EXAMINING METHODS AND PROCEDURES

No new examining methods and procedures were used for 2018/19.

C. CHANGES IN EXAMINING METHODS AND PROCEDURES CURRENTLY UNDER DISCUSSION OR CONTEMPLATED FOR THE FUTURE

No changes are under discussion for 2019/20.

D. NOTICE OF EXAMINATION CONVENTIONS FOR CANDIDATES

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and examination conventions in full are on-line at https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. GENERAL COMMENTS ON THE EXAMINATION

Timetable

The examinations began on Monday 24th June at 2.30pm and ended on Friday 28th June at 12:30pm.

Mitigating Circumstances Notices to Examiners

A sub-set of the Examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of circumstances for each factors affecting performance application received. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further detail.

Determination of University Standardised Marks

For the papers that are common with Mathematics, the examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM), and used the same scaling functions as applied for candidates in Mathematics.

Recommendations for Next Year's Examiners and Teaching Committee

There are no recommendations specific to Mathematics & Philosophy. General recommendations are made in the report on the Preliminary Examination in Mathematics.

B. EQUAL OPPORTUNITIES ISSUES AND BREAKDOWN OF THE RESULTS BY GENDER

The breakdown of the final classification by gender is as follows:-

Table 2: Breakdown of results by gender

Class	Number								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	0	7	7	1	5	6	2	2	4
Pass	6	5	11	4	3	7	5	8	13
Partial Pass	2	0	2	1	0	1	0	0	0
Fail	0	0	0	0	0	0	0	0	0
Total	8	12	20	6	8	14	7	10	17

Class	Percentage								
	2019			2018			2017		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	0	58.33	35	16.67	62.5	42.86	28.57	20	25.53
Pass	75	41.67	55	66.67	37.5	50	71.43	80	76.47
Partial Pass	25	0	10	16.67	0	7.14	0	0	0
Fail	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. STATISTICS ON CANDIDATES' PERFORMANCE IN EACH PART OF THE EXAMINATION

Mathematics I

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	11.00	3.80	10.80	2.73
Q2	14.44	3.83	14.75	2.59
Q3	14.00	3.30	13.91	4.00
Q4	12.05	4.81	12.81	4.07
Q5	13.00	4.47	11.63	4.20
Q6	14.54	4.22	13.76	3.88
Q7	12.53	4.73	13.00	3.22

Mathematics II

	Maths and Philosophy		Single School	
Question	Mean	Std Dev	Mean	Std Dev
Q1	10.00	3.02	11.40	3.16
Q2	16.00	5.19	14.31	3.57
Q3	10.88	4.39	11.77	3.15
Q4	11.00	3.35	10.95	3.64
Q5	10.63	5.78	14.02	5.04
Q6	11.20	5.85	11.68	4.82
Q7	8.05	4.17	8.48	3.77

Mathematics III(P)

Question	Maths and Philosophy		Single School	
	Mean	Std Dev	Mean	Std Dev
Q1	12.71	5.65	16.61	3.11
Q2	10.11	4.54	13.83	3.87
Q3	10.93	5.51	15.57	4.41
Q4	6.25	5.43	8.48	5.33
Q5	12.60	3.89	11.24	4.07
Q6	10.42	4.08	12.05	3.85

Elements of Deductive Logic

AvgUSM	StdDevUSM
70.3	16.25

Introduction to Philosophy

AvgUSM	StdDevUSM
63.9	4.9

D. COMMENTS ON PAPERS AND ON INDIVIDUAL QUESTIONS

See the Mathematics report for reports on the following papers:

Mathematics I

Mathematics II

Mathematics III(P)

Report on Elements of Deductive Logic

This report on the EDL paper covers students in Computer Science & Philosophy, Maths & Philosophy, and Physics & Philosophy.

Comments on single questions

Question 1 (39 answers, mean 18.71, SD 4.58)

This question, which concerned duality in the language $\mathcal{L}_1^{++}$ with $\top$, $\bot$, and an n-ary connective for every n-ary truth function, was the most popular on the examination, and there were many strong answers.

Part (a) was intended to be an easy introduction to the issues, and most students had no difficulties with it. A few students made the small error of forgetting to specify cases for $\top$ and $\bot$ as well as the n-ary connectives in inductive definitions. A very small number misunderstood the question entirely, providing only clauses for the $\mathcal{L}_1$ connectives rather than for every connective in $\mathcal{L}_1^{++}$ (and made similar errors in the other sections); this was presumably due to failure to read the question carefully.

Similarly, part (b) posed few problems. Serious errors arose only very occasionally in (b)(ii). No substitution map $P_1 \wedge \neg P_2 \mapsto (P_1 \vee P) \wedge (\neg P_2 \vee P)$ exists. There are substitution maps taking $P_1 \wedge \neg P_2$ to a sentence *logically equivalent* to $(P_1 \vee P) \wedge (\neg P_2 \vee P)$—but this is not what the question asks for. A few students provided such sentences as purported positive answers to (b)(ii).

Parts (c) and (d) represent the core of the question. There were very few problems with (c)(i). A number of students were insufficiently explicit in (c)(ii): the fact that a tautology is true in every structure should be clearly stated, for it is this that allows one to move from $\mathcal{A} \vDash \phi$ to $\mathcal{A}^\pi \vDash \phi$. Subparts (d)(i) and (d)(ii) are conceptually straightforward, although (d)(ii) is tedious to write out, and some students had run out of time or abandoned the question by this point. The simplest solution to (d)(iii) involves manipulating definitions to show that $|\gamma|_{\mathcal{A}^*} = |\gamma|_{\mathcal{A}^*}$ for γ a sentence letter and then using the Relevance Lemma; some students gave correct, but more complicated, inductive proofs in which these key ideas were applied in a different order.

Question 2 (34 answers, mean 17.16, SD 4.59)

This question was popular and, for the most part, posed few conceptual problems. Part (a) required only knowledge of definitions. For part (b), the simplest answers result from mechanically applying the DNF theorem: in (b)(i), for example, $(P_1 \wedge \neg P_2 \wedge \neg P_3) \vee (\neg P_1 \wedge P_2 \wedge \neg P_3) \vee (\neg P_1 \wedge \neg P_2 \wedge P_3)$ defines g. Other solutions are possible, but a few students appear to have made errors seeking needlessly elegant or compressed sentences.

Almost all students who failed to answer (c)(i) and (c)(ii) adequately made the same error: they proved only the 'hints' provided in the questions, without explaining how to get from there to proofs of the general result. For instance, (c)(i) asks the reader to prove that the connective G representing the truth function g defined in (b)(i) is not, on its own, expressively adequate; the hint reads 'First note that $g(0,0,0) = 0$'. Establishing the hint is a simple calculation, and it shows the way to the idea of the proof: no ternary truth function f with the property that $f(0,0,0) = 1$ can be represented using G alone.

But this must be proved explicitly by a induction on complexity of formulas. The induction, of course, is very easy: Let $\mathcal{G}$ be an arbitrary structure with $|P_1|_{\mathcal{G}} = |P_2|_{\mathcal{G}} = |P_3|_{\mathcal{G}} = 0$. Clearly if ϕ is atomic, then $|\phi|_{\mathcal{G}} = 0$. The induction hypothesis is that, for all ψ of complexity $< n$, $|\psi|_{\mathcal{G}} = 0$. Now we can use the fact that $g(0,0,0) = 0$ and the fact that any sentence ϕ must have the form $\mathsf{G}(\psi_1, \psi_2, \psi_3)$ for ψ_1, ψ_2, and ψ_3 of complexity less than that of ϕ to show that $|\phi|_{\mathcal{G}} = 0$ for any ϕ of complexity n. So $|\phi|_{\mathcal{G}} = 0$ for all ϕ, and no such sentence can define an f where $f(0,0,0) = 1$.

Subpart (c)(ii) follows precisely the same idea. For (c)(iii), it is simply a question of finding translations using creative syntactic manipulation. In general, students who struggled here either simply ran out of time or made calculational mistakes. Many answers are possible: translating $\neg\phi$ by $\mathsf{H}(\phi, \phi, \phi)$ and $\phi \wedge \psi$ by $\mathsf{G}(\phi, \mathsf{H}(\psi, \psi, \psi), \mathsf{H}(\psi, \psi, \psi))$, and then applying the de Morgan equivalence for $\vee$, is probably the most obvious.

Question 3 (32 answers, mean 18.31, SD 5.70)

This question had more near-perfect scores, and more significantly erroneous scripts, than others.

Part (a) was purely definitional. In (b), a few students made confused appeals to the Relevance Lemma, failing to note that the demonstrandum simply is a formulation of the Relevance Lemma. More commonly, tiny mistakes occurred in the formulation of the base case of the induction: it should be explicitly noted that where the sentence is a sentence letter, it must by definition be an α_i for some $1 \leq i \leq n$.

Most students who struggled seriously with the question did so with (c). The most common error in (c)(i) was failing to recognize that there are two cases to be considered. In the base case, for instance, either (1) there are infinitely many $\mathcal{A} \in \mathbf{S}$ such that $|\alpha_1|_{\mathcal{A}} = \mathrm{T}$ or (2) there are not. In case (1), it holds immediately that $|\alpha_1|_{\mathcal{A}_\mathbf{S}} = \mathrm{T}$ and thus there are infinitely many $\mathcal{A} \in \mathbf{S}$ such that $\mathcal{A}$ and $\mathcal{A}_\mathbf{S}$ agree up to α_1. In case (1), there are only finitely many $\mathcal{A} \in \mathbf{S}$ such that $|\alpha_1|_{\mathcal{A}} = \mathrm{T}$; since $\mathbf{S}$ is infinite, there must be infinitely many $\mathcal{A} \in \mathbf{S}$ such that $|\alpha_1|_{\mathcal{A}} = \mathrm{F}$. But, by definition, $|\alpha_1|_{\mathcal{A}_\mathbf{S}} = \mathrm{F}$, so there are infinitely many $\mathcal{A} \in \mathbf{S}$ such that $\mathcal{A}$ and $\mathcal{A}_\mathbf{S}$ agree up to α_1. Many students overlooked the possibility of (2). (A similar bifurcation of cases occurs in the induction step.) Students making this mistake received only half credit.

Most students who successfully completed (c)(i) found (c)(ii–iv) fairly straightforward. The only moderately tricky point is the key step in (c)(iii): noticing that $|\alpha_j|_{\mathcal{A}_i} = \mathrm{T}$ when $j \geq i$. When they occurred, poor results on (c)(iii–iv) tended to result from running out of time or proofs that exhibited general structural confusion and obscurity rather than any one stereotypical error.

Question 4 (25 answers, mean 17.10, SD 4.57)

Part (a) of this question posed few difficulties. In part (b), almost all students realized that $\{P\}$ is a minimal interpolant set, but many forgot to prove it or gave inadequate proofs. It suffices to show that every element of the interpolant set must be equivalent to one of $\{P, \neg P, P \vee \neg P, P \wedge \neg P\}$, and among those only P is an interpolant for $P \wedge R \vDash P \vee Q$.

Essentially the same strategy is required in part (c), although now there are sixteen cases to check (corresponding to sentences containing P_1 and P_2 defining the sixteen binary truth functions). Very few students correctly found all the (equivalence classes of) interpolants: $P_1, P_1 \wedge P_2, P_1 \leftrightarrow P_2, P_2 \rightarrow P_1$, generally due to calculational errors or oversights, but all who attempted the part received significant credit.

In part (d), most students understood the key point: where ϕ and ψ are $\mathcal{L}_2$ sentences that share no predicate letters, there need be no interpolant, since $\mathcal{L}_2$ does not contain $\top$ and $\bot$. (Thus, for example, there is no interpolant for $Fa \wedge Fb \vDash Gc \vee \neg Gc$.) Some students thought the problem could be evaded by using purely logical sentences with equality, which would be correct for

$\mathcal{L}_=$ but is inapposite here.

Question 5 (16 answers, mean 17.88, SD 3.52)

Few students had any difficulty with part (a), which merely required paying attention to the stated syntax rules for the language ('Klungon') under consideration. There was one very common error in part (b). The syntax rules are stated as inductive clauses; the question asks the student to provide an explicit characterization and prove the equivalence. (The explicit characterization is $\{x \text{ is a string } : x = \alpha_1{}^\frown \cdots {}^\frown \alpha_n{}^\frown a^\frown \beta_1{}^\frown \cdots {}^\frown \beta_n \text{where } \alpha, \beta \in \{a, b, c\}\}$). Call this set S. To demonstrate the equivalence, the student must prove both that every string in S falls under the inductive definition and that every string that falls under the inductive definition is in S. Only a minority of students remembered to carry out both of these tasks.

In part (c), few students had serious trouble with (i) or (ii). In subpart (iii), the nonexistence of ϕ such that $\Gamma \vDash \phi$ for all Γ follows immediately from the fact that, for any ϕ, there exists some valuation v_f and sentence ψ such that $v_f(\psi) < v_f(\phi)$, which in turn follows from the existence of a valuation v_f such that $\{v_f(\phi) : \phi \text{ is a Klungon wff}\}$ is unboundedly small. Many students realized this but failed to prove the existence of such a valuation (which is not difficult: the example I-function $f(a) = 0, f(b) = -1, f(c) = 2$ suffices, as $v_f(a) = 0$ and $v_f(a^\frown \alpha^\frown c) < v_f(\alpha)$. Similar considerations show that it is not the case that i $\Gamma \vDash \phi$, then $\Gamma \cup \{\psi\} \vDash \phi$ for all ψ. In subpart (iv), it suffices to show that any ϕ with the property $\Gamma \vDash \phi$ must have $v_f(\phi) = 0$ for all v_f. There was no one characteristic type of error in this responses, although many students failed to complete the subpart.

Question 6 (8 answers, mean 19.19, SD 4.67)

This was the least popular question, but virtually all responses were extremely good. None of the material in this question goes beyond the contents of *The Logic Manual*, although correct synthesis requires some ingenuity. In parts (a)–(c) errors, if present, were generally technical (oversights, misapplication of rules, or ineffective strategy in the more complicated natural deduction proofs). Some students merely displayed ND_2 derivations of $\forall$Intro* and $\forall$Elim* (in (c)(i)) or ND_2^* derivations of $\forall$Intro and $\forall$Elim (in (c)(ii)) without explaining how this yields the result. This can be done in a brief sentence, but some elaboration is necessary for a proper proof. In (d), there are numerous ways to prove the result: the simplest to note is that, if ψ is logically true, then $\Gamma \cup \{\psi\} \vDash \phi$ if and only if $\Gamma \vDash \phi$ and then apply Completeness to show the admissibility of the proposed rule in ND_2. All of the responses displayed a grasp of the general idea, although some had unclarities and lacunae that led to less than full credit.

Question 7 (14 answers, mean 14.88, SD 5.88)

Most students who attempted this question completed it well; there were a few extremely poor responses to parts (b) and (c), however, which caused it to have the lowest mean and highest standard deviation overall of all the questions. It is difficult to attribute specific localized errors to those responses; instead, they simply displayed a fundamental misunderstanding of the nature of $\mathcal{L}_2$ assignments. (More than one student, for instance, forgot that that open formulae such as Fx need to be included in the base cases of (b)(ii–iii); those students also failed to give anything like a reasonable treatment of substitution in (b)(iii).) Part (c) follows straightforwardly from (a)(ii), (b)(iii), the semantc clause for the universal quantifier, and a bit of manipulation. Some students who gave otherwise adequate answers failed to attempt or complete it, but among those who gave full responses it posed difficulty only for those students who displayed the kind of conceptual difficulties in (b)(ii–iii) discussed above.

Question 8 (14 answers, mean 15.14, SD 5.21)

This question had some poor responses, but no specific characteristic types of error. Parts (a) and (b)(i–ii) posed no conceptual difficulties, but many students failed to produce a correct natural deduction proof or made serious formalization errors. For (b)(iii), some students failed to realize that, since C must be $\neg\exists x(Q_0x \land \forall y(Ryx \leftrightarrow Py))$ or something similar, it cannot be entailed by $\{B_n : n > 0\}$, none of whose instances contain Q_0. Part (c) is a fairly straightforward use of Compactness: students who made it this far had little trouble with it, although many failed to complete (or even attempt) the part.

Report on Introduction to Philosophy

General Philosophy Questions

Question 1a (5 answers). Good answers often applied familiar theories of knowledge to determine what a BIV may know, sometimes bringing in considerations from Descartes. Weaker answers sometimes took this as an invitation to discuss scepticism more generally. A surprising number of candidates seemed to overlook the obvious point that, prima facie at least, most of the BIV's beliefs fail to constitute knowledge because they are false.

Question 1b (10 answers). This popular question was generally well done, with many answers showing a good knowledge of the Gettier literature, and addressing the question systematically. The best answers often also engaged critically with the suggestion in the question that because true belief is necessary there must be some X where true belief + X is necessary and sufficient.

Question 2a (1 answer).

Question 2b (5 answers). This question wasn't so well done. Many answers tended towards surveying the problem of induction rather than attempting to engage with the question posed, with only a minority tackling the issue of whether there is vicious circularity in using deductive reasoning to justify deduction.

Question 3a (3 answers).

Question 3b (8 answers). Another popular question, that almost always received answers that displayed a good level of knowledge of the relevant literature, and engaged with the question posed.

Question 4a (0 answers).

Question 4b (1 answer).

Question 5a (2 answers).

Question 5b (4 answers).

Question 6a (1 answer).

Question 6b (3 answers).

Frege: Foundations of Arithmetic

Question 7 (4 answers).

Question 8 (1 answer).

Question 9 (10 answers). The quality of answers to this popular question varied. Good answers displayed a firm grasp of Frege's object–concept distinction, and engaged with his arguments for not taking numbers to be second-level concepts. Weaker answers sometimes misrepresented Frege's views, or were thin on argument. In this question, as in some of the others, candidates sometimes exhibited use–mention confusion.

Question 10 (10 answers). This question also received answers of variable quality. Good answers gave a clear account of Frege's definition, and took a reasoned view as to whether or not it avoided the Caesar problem and whether or not it needed to. Weaker answers sometimes failed to give a clear and correct account of Frege's definition. In particular, some candidates incorrectly conflated the number belonging to F with the extension of F.

Question 11 (5 answers). This question received some very strong answers that were able to accurately summarize Frege's proof strategy, and identify points where he draws on the thesis that numbers are objects. Good answers often explained the need for the alternative (Russellian) treatment, which takes numbers to be second-level concepts, to assume that there are infinitely many objects at the bottom of the type hierarchy.

Question 12 (6 answers). This question was, on average, the best done in the Frege section. Answers generally showed a good understanding of relevant literature surrounding neo-Fregeanism. Candidates often did a good job of identifying a special epistemic or semantic status arguably both (i) possessed by Hume's Principle and (ii) not possessed by Peano Arithmetic; sometimes, however, they failed to sufficiently interrogate the second claim.

E. COMMENTS ON PERFORMANCE ON IDENTIFIABLE INDIVIDUALS

This section has been redacted from the public report.

F. NAMES OF MEMBERS OF THE BOARD OF EXAMINERS

- Prof. Dmitry Belyaev (Chair for Preliminary Examinations)
- Dr Adam Caulton
- Dr Richard Earl
- Prof. James Studd