UNIVERSITY OF OXFORD

Discontinuities and Singularities in Nonlinear Evolution PDEs

7-8 October 2024

The H B Allen Centre, 25 Banbury Road, Oxford

Oxford Centre for Nonlinear PDE

UKRI Engineering and Physical Sciences Research Council

VENUE

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VENUE

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PROGRAMME

Monday, 7 October

9:30	**Welcome remarks**
9:35	Gigliola Staffilani, MIT
10:20	Eduard Feireisl, Czech Academy of Sciences
11:05	BREAK
11:35	Giuseppe Savare, Università Bocconi
12:20	Mark Peletier, Eindhoven University of Technology
13:00	LUNCH
14:05	Michele Coti-Zelati, Imperial College London
14:50	Lucia Scardia, Heriot-Watt University
15:30	BREAK
16:00	Mahir Hadzic, University College London
16:45	Edriss Titi, University of Cambridge
17:25	Closing remarks

PROGRAMME

Tuesday, 8 October

9:00	**Welcome remarks**
9:10	Benoît Perthame, Sorbonne Université
9:55	Federica Dragoni, Cardiff University
10:40	BREAK
11:10	Mikhail Korobkov, Fudan University, Shanghai
11:55	David Bourne, Heriot-Watt University
12:40	LUNCH
13:45	Simon Schulz, Scuola Normale Superiore di Pisa
14:30	Denis Serre, ENS de Lyon
15:15	Qian Wang, University of Oxford
16:00	Closing remarks and optional refreshments

ABSTRACTS

David Bourne, Heriot-Watt University
Optimal transport theory and the compressible semi-geostrophic equations

The semi-geostrophic equations are a simplified model of large-scale atmospheric flows and frontogenesis. In this talk I will discuss existence and numerical approximation of weak solutions of the semi-geostrophic equations for a compressible fluid. This is joint work with Charlie Egan (Göttingen), Théo Lavier and Beatrice Pelloni (Heriot-Watt), and Quentin Mérigot (Université Paris-Saclay).

Michele Coti-Zelati, Imperial College London
Stability thresholds in three dimensional stratified fluids

The stability of shear flows in the fluid mechanics is an old problem dating back to the famous Reynolds experiments in 1883. The question is to quantify the size of the basin of attraction of equilibria of the Navier-Stokes equations depending on the viscosity parameters, giving rise to the so-called stability threshold. In the case of a three-dimensional homogeneous fluid, it is known that the Couette flow has a stability threshold proportional to the viscosity, and this is sharp in view of a linear instability mechanism known as the lift-up effect. In this talk, I will explain how to exploit stratification (i.e. non-homogeneity in the fluid density) to improve this bound: the coupling between density and velocity gives rise to oscillations, which suppress the lift-effect. This can be captured at the linear level in an explicit manner, and at the nonlinear level by combining sharp energy estimates with suitable dispersive estimates.

Federica Dragoni, University of Cardiff
Semiconcavity of the square distance in Carnot groups.

Semiconcavity and semiconvexity are key regularity properties for functions with many applications in a broad range of mathematical subjects. The notions of semiconcavity and semiconvexity have been adapted to different geometrical contexts, in particular in sub-Riemannian structures such as Carnot groups, where they turn out to be extremely useful for the study of solutions of degenerate PDEs. In this talk I will show that, for a suitable class of Carnot groups, the Carnot-Carathéodory distance is semiconcave, in the sense of the group,

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in the whole space. I will also give some applications to solutions of non-coercive Hamilton-Jacobi equations. Joint work with Qing Liu and Ye Zhang from OIST (Okinawa, Japan).

Eduard Feireis, Institute of Mathematics, Czech Academy of Sciences
On conditional regularity for compressible fluid models

We discuss conditional regularity/blow up criteria for some models of compressible viscous and heat conducting fluids. In particular, the so-called Nash conjecture is shown. Application to numerical analysis of problems with uncertain data are presented.

Mahir Hadzic, University College London
On stable implosion for self-gravitating fluids

We first review the existence theory of radial self-similar imploding solutions to the compressible Euler-Poisson and the Einstein-Euler system, representing collapsing stars. They are known in the literature as the Larson-Penston and the relativistic Larson Penston solutions respectively.

We then explain the proof of nonlinear stability of the (Newtonian) Larson-Penston solution against radial perturbations. At the heart of the proof is a difficult non self-adjoint spectral problem and a new high-order energy framework which crucially takes advantage of the various monotonicity properties of the Larson-Penston self-similar profile. This is joint work with Yan Guo, Juhi Jang, and Matthew Schrecker.

Mikhail Korobkov, Fudan University, Shanghai
Classical Leray Problems on Steady–State Navier–Stokes system: recent advances and new perspectives

In recent years, using the geometric and real analysis methods, essential progress has been achieved in some classical Leray’s problems on stationary motions of viscous incompressible fluid: the existence of solutions to a boundary value problem in a bounded plane and three-dimensional axisymmetric domains under the necessary and sufficient condition of zero total flux; the uniqueness of the solutions to the plane flow around an

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obstacle problem in the class of all D-solutions, the nontriviality of the Leray solutions (obtained by the "invading domains" method) and their convergence to a given limit at low Reynolds numbers; and, more generally, the existence and properties of D-solutions to the boundary value problem in exterior domains in the plane and three-dimensional axisymmetric case, etc. A review of these advances and methods will be the focus of the talk. Most of the reviewed results were obtained in our joint articles with Konstantin Pileckas, Remigio Russo, Xiao Ren, and Julien Guillod, see, e.g., the recent survey paper J. Math. Fluid Mech. 25 (55) (2023).

Mark Peletier, Technische Universiteit Eindhoven
Noisy dislocations: 2D Coulomb interaction with Brownian noise

Some Keller-Segel PDEs can be viewed as the many-particle limit of a system of stochastic particles that attract each other. The particles experience Brownian noise, and in the critical case of Coulomb attraction the interplay between attraction and noise gives rise to highly non-trivial behavior, as was shown by Fournier, Jordain, and Tardy.
In this paper we change the microscopic particle system from all-pair attraction to a signed attraction-repulsion, in which opposite signs attract and equal signs repel each other. This is inspired both by colloids that can be electrically charged and by 2D dislocation models that have similar `charges'. In both cases the interaction again has Coulomb scaling and is therefore critical.
We show that for this `electronic' attraction-repulsion setup the situation is different than for the all-attracting case, and we give a detailed description of collision and de-collision events. We show that for this type of interaction the many-particle limit is well posed for interaction strengths that grow faster with N than in the case of all-attracting particles. In particular, the many-particle case is well-posed in the trivial-scaling case of `zooming out' while preserving total particle charge.
Finally, we establish a many-particle, mean-field limit to a Keller-Segel-type PDE. This is joint work with Thomas Slangen (Eindhoven) and Patrick van Meurs (Kanazawa, Japan).

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Benoît Perthame, Sorbonne-Université
Adaptive evolution and concentrations in parabolic PDEs

Living systems are characterized by variability; in the view of C.~Darwin, they are subject to constant evolution through the three processes of population growth, selection by nutrients limitation and mutations.

Several mathematical theories have been proposed in order to describe the dynamics generated by the interaction between their environment and the trait selection of the `fittest'. One can use stochastic individual based models, dynamical systems, game theory considering traits as strategies. From a populational point of view, the population obeys an integro-partial-differential equation for the density number.

We will give a self-contained mathematical model of such dynamics and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the fittest trait and eventually to compute various forms of branching points which represent the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the above mentioned parabolic equations once appropriately rescaled. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that describe the population. We will show that a new type of Hamilton-Jacobi equation, with constraints, naturally describes this asymptotic.

Recent developments concern non-proliferative advantages and lead to define the notion of `effective fitness'.

Giuseppe Savare, Università Bocconi
The Derrida-Lebowitz-Speer-Spohn equation: variational approaches and structure-preserving discretizations.

We will discuss different variational structures for the fourth-order nonlinear DLSS equation, in particular a novel gradient flow formulation in terms of a metric generalizing martingale

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transport. In the simpler 1D case with periodic boundary conditions, we propose a spatial discretization that preserves these intriguing structures: as a by-product, we obtain discrete versions of relevant nonlinear functional inequalities, which are useful to pass to the limit in the discretization and to recover a weak solution.
(In collaboration with Daniel Matthes, Eva-Maria Rott, and André Schlichting.)

Lucia Scardia, Heriot-Watt University
Shape Optimisation for nonlocal anisotropic energies

We consider shape optimisation problems for sets of prescribed mass, where the driving energy functional is nonlocal and anisotropic. More precisely, we deal with the case of attractive/repulsive interactions in two and three dimensions, where the attraction is quadratic and the repulsion is given by an anisotropic variant of the Coulomb potential. Under the sole assumption of strict positivity of the Fourier transform of the interaction potential, we show the existence of a threshold value for the mass above which the minimiser is an ellipsoid, and below which the minimiser does not exist. If, instead, the Fourier transform of the interaction potential is only nonnegative, we show the emergence of a dichotomy: either there exists a threshold value for the mass as in the case above, or the minimiser is an ellipsoid for any positive value of the mass.
This is joint work with Riccardo Cristoferi and Maria Giovanna Mora.

Simon Schulz, Scuola Normale Superiore
The Morawetz problem for supersonic flow with cavitation

We present a novel existence result for supersonic flows around an obstacle of a general shape, doing so by means of the vanishing viscosity method and compensated compactness, as per the original approach of C.S. Morawetz. Our method involves a careful asymptotic analysis of the entropies of the steady compressible potential flow system in the vicinity of the vacuum. These entropies satisfy, in the most general setting, a mixed elliptic-hyperbolic equation with an irregular singularity at cavitation. This is joint work with Gui-Qiang Chen and Tristan Giron.

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Denis Serre, Ecole Normale Supérieure de Lyon

Compensated Integrability in bounded domains ; Applications

Compensated Integrability is a tool from Functional Analysis to study energy-momentum tensors A in Mathematical Physics, especially that of either Gas or Hard Spheres dynamics [1,2,3]. In practice, it provides a space-time estimate of quantities which are dominated neither by the mass density, nor by the energy density. So far, C.I. was well understood only in the space R^n. The application to the Euler equation in R^d led to the estimate

$$\int_0^{+\infty} dt \int_{R^d} \rho^{\frac{1}{d}} p \, dy \le c_d M^{\frac{1}{d}} \sqrt{ME} \, .$$

However the applications to evolution problems in bounded domains with natural boundary conditions led to unsatisfactory results, the estimate deteriorating near the boundary [4]. We show for the first time that C.I. is efficient in bounded domains, provided that the tangential part of the normal trace $A\vec{v}$ is controlled in the space of bounded measures [5]. This applies successfully to the Euler system with the slip boundary condition $u \cdot \vec{v} = 0$, as well as to Hard Spheres dynamics with specular reflection. The new estimates are established over bounded time intervals (they would fail over R_+). Somehow, they give an estimate of a space integral, for instance of

$$\int_{R^d} \rho^{\frac{1}{d}} p \, dy$$

for gas dynamics, “in the mean" with respect to time.

Gigliola Staffilani, MIT

A curious phenomenon in wave turbulence theory

In this talk we will use the periodic cubic nonlinear Schrödinger equation to present some estimates for the long time dynamics of the energy spectrum, a fundamental object in the study of wave turbulence theory. Going back to Bourgain, one possible way to conduct the analysis is to look at the growth of high Sobolev norms. It turns out that this growth is sensitive to the nature of the space periodicity of the system. I will present a combination of old and very recent results in this direction.

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Edriss S. Titi, University of Cambridge
Inviscid Voigt Regularization of Fluid Models: Blow-up Criterion, Computations and Statistical Properties

In this talk we will report the results of a computational investigation of a new blowup criterion for the 3D incompressible Euler equations, which does not rely on the seminal Beale-Kato-Majda blow-up criterion. This criterion is based on an inviscid regularization of the Euler equations known as the $3D$ Euler-Voigt equations, which are known to be globally well-posed. Moreover, simulations of the $3D$ Euler-Voigt equations also require less resolution than simulations of the $3D$ Euler equations for fixed values of the regularization parameter $\alpha > 0$. Therefore, the new blow-up criteria allow one to gain information about possible singularity formation in the $3D$ Euler equations indirectly, namely by simulating the better-behaving 3D Euler-Voigt equations. The new criterion is only known to be sufficient criterion for blow-up. Therefore, to test the robustness of the inviscid-regularization approach, we also investigate analogous criteria for blow-up of the $1D$ Burgers equation, where blow-up is well known to occur. Notably, the Voigt inviscid regularization approach applies equally to other hydrodynamical models, and it can be shown that its solutions converge, as the regularization parameter $\alpha \to 0$, to the corresponding solutions of the underlying hydrodynamical model for as long as the latter exist.
Furthermore, if time allows, we will also report about the effect of the Voigt regularization on the statistical properties of turbulent solutions.

Qian Wang, Mathematical Institute, University of Oxford
On global dynamics of 3-D irrotational compressible fluids

We consider global-in-time evolution of irrotational, isentropic, compressible Euler flow in 3-D, for a broad class of H^4 classical Cauchy data without assuming symmetry, prescribed on an annulus surrounded by a constant state in the exterior. By giving a sufficient condition on the initial data and using the nonlinear structure of the compressible Euler equations, we construct global exterior solutions in H^4 for the broad class of data, with a rather general subclass forming rarefaction at null infinity. Our result does not require smallness on the transversal derivatives of classical data, thus applies to data with a total energy of any size. This is on my recent work arXiv:2407.13649.

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