



Isaac Newton Institute for Mathematical Sciences

Visitor Programme at University of Oxford

Thursday 22nd September 2022

<p><u>10:30 – 11:00</u></p>	<p>Welcome and Refreshments - <i>Mezzanine, Mathematical Institute</i> (GPS Location)</p> <p>Prof. Gui-Qiang G. Chen (University of Oxford) Website LinkedIn</p> <p>Prof. Endre Süli (University of Oxford) Website</p> <p>Prof. Michael Shearer (North Carolina State University) Website</p>
<p><u>11:00 – 12:30</u></p>	<p>3 Talks to introduce Dispersive Hydrodynamics - <i>L5, Mathematical Institute</i> (GPS Location)</p> <p>Prof. Mark Hoefer (University of Colorado, Boulder, USA) Website LinkedIn</p> <p>Viscous Core-Annular Flows: A Laboratory Playground for Dispersive Hydrodynamics</p> <p><i>Dispersive Hydrodynamics is a mathematical framework to describe multiscale nonlinear wave phenomena in dispersive media, including both dynamic and stochastic aspects of wave propagation. With a variety of physical applications, it is a vibrant and developing field. This talk will present experiments with a special core-annular flow—two viscous, miscible fluids with high viscosity contrast, one rising buoyantly within the other—in order to highlight a rich variety of dispersive hydrodynamic phenomena including solitons, dispersive shock waves, breathers, and soliton gases. Along the way, Whitham modulation theory will be used to help describe the experimental results.</i></p> <p>Prof. Annalisa Calini (College of Charleston, North Carolina, USA) Website LinkedIn</p> <p>Knotted solutions of the Vortex Filament Equation without self-crossings</p> <p><i>The vortex filament equation (VFE) or binormal flow—a localized induction approximation of the Biot-Savart law—describes the self-induced dynamics of a vortex filament in a 3-dimensional ideal fluid. Its connection with the cubic focusing Nonlinear Schrödinger Equation (NLS) through the Hasimoto map allows the use of tools from soliton theory to construct large classes of solutions. However, most such solutions will exhibit self-crossings, thus ceasing to represent physical filaments at some finite time. In this talk I will focus on the construction of a family of knotted vortex filaments coming from finite-genus solutions of the NLS, including torus and cable knots, whose knot types do not change throughout the VFE evolution. If time permits, I will comment on their linear stability properties.</i></p> <p>Prof. Patrick Sprenger (University of Cambridge, UK) Website LinkedIn</p> <p>Dispersive shock waves and traveling wave solutions of fifth order KdV equations</p> <p><i>Dispersive shock waves (DSWs) are ubiquitous wave patterns that arise from gradient catastrophe in dispersive hydrodynamic systems. A canonical model of weakly nonlinear dispersive waves is the Korteweg-de Vries equation, where DSWs are modeled as a modulated, nonlinear wavetrain. This coherent, nonlinear structure is described via a continuous, rarefaction solution of the Whitham modulation system. In contrast, when a fifth order linear dispersive term is included with, or replaces the third order linear dispersion term in the KdV equation, the resulting DSWs are quite different. In particular, this system admits DSW solutions that are comprised of a traveling wave portion that is described via a discontinuous shock solution of the Whitham modulation equations. This talk focuses on such traveling wave solutions whose parameters satisfy the Whitham Rankine-Hugoniot jump relations. Focus will be given on the computation of these solutions via the intersection of invariant manifolds of two periodic orbits that solve a fourth order Hamiltonian dynamical system.</i></p>

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<u>12:45</u> – <u>14:15</u>	Lunchtime - <i>Brasserie Blanc</i> (GPS Location)
<u>14:30</u> – <u>15:30</u>	<p>2 Talks - L5, Mathematical Institute (GPS Location)</p> <p>Dr. Antonio Moro (Northumbria University, Newcastle, UK) Website</p> <p>Integrability and complexity in statistical mechanics: thermodynamic limit vs viscous / dispersive regularisation</p> <p><i>The theory of integrable nonlinear conservation laws arises as a ‘universal’ paradigm for the description and classification of phase transitions, cooperative and catastrophic behaviours in many body systems at the crossroad of integrable systems, statistical mechanics and random matrix theory.</i></p> <p><i>A key element of this paradigm is the construction of suitable differential identities for partition functions from which one can deduce nonlinear partial differential equations, typically a hierarchy of hydrodynamic conservation laws, for the order parameters of the theory. Critical phenomena and phase transitions are therefore understood in terms of asymptotic properties of the solutions in the low viscosity/weak dispersion regime for these equations.</i></p> <p><i>We illustrate, via specific examples, how viscosity underpins the occurrence of phase transitions in “simple” systems while dispersion arises as a possible mechanism for the description of emergent complex behaviours and out of equilibrium thermodynamics..</i></p> <p>Prof. Ted Johnson (University College London, UK) Website LinkedIn</p> <p>Dispersive Shock Waves in Coastal Flows</p> <p><i>Coastal or boundary currents are an integral part of global ocean circulation. For example, currents may respond to external forcing or intrinsic instability by expelling vortex filaments or larger eddies into the ocean, with implications for the mixing of coastal and ocean waters; and currents driven by outflows are important for the transport of freshwater, pollutants and land-derived nutrients. There is also much interest in the behaviour of ‘free’ fronts, i.e. those that are far from the coast, which can be used to model western boundary currents such as the Gulf Stream or the Kuroshio Extension. In the limit of rapid rotation the governing equations reduce to the quasi-geostrophic equations - a modified form of the two-dimensional Euler equations. For the problems considered here the vorticity of the flow is unity within the current and zero elsewhere. The unapproximated solution can thus be obtained numerically to high accuracy by applying the method of Contour Dynamics to the development of the current-ocean interface. These solutions provide comparisons for estimating the accuracy of asymptotic solutions.</i></p> <p><i>Alongshore variations in the flows take place over scales large compared to offshore scales and so analysis of the flows leads naturally to a long-wave equation for the current- ocean interface. Two examples will be discussed in depth. First, the development of the flow when fluid is discharged from a source on the coast to turn and form an alongshore current (Johnson et al. 2017) and, second, the Riemann problem for the subsequent development of a step change in width of a coastal flow (Jamshidi & Johnson 2020). The flux function appearing in the long-wave equation is non-convex and this leads to a wide variety of behaviours.</i></p>
<u>15:45</u> – <u>18:00</u>	Afternoon Tea - <i>Worcester College</i> (GPS Location)
<u>18:00</u>	END

More details can be found at: <https://www.maths.ox.ac.uk/node/60185>