Handbook for the Undergraduate Mathematics Courses
Supplement to the Handbook
Honour School of Mathematics
Syllabus and Synopses for Part A 2006–7
for examination in 2007

Contents

1 Foreword

Notice of misprints or errors of any kind, and suggestions for improvements in this booklet should be addressed to the Academic Assistant in the Mathematical Institute.

1.1 Honour School of Mathematics

[See the current edition of the *Examination Regulations* for the full regulations governing these examinations.]

In Part A each candidate shall be required to offer the 4 written papers from the schedule of papers for Part A (below).

Part A shall be taken on one occasion only (there will be no resits). At the end of the Part A examinations, a candidate will be awarded four 'University Standardised Marks' (USMs) for their performance in Part A. These will be carried forward into the classification awarded at the end of the degree. In this classification, the marks in Part A will be given a 'weighting' of 2, and the marks in Part B will be given a 'weighting' of 3. All students who complete the first three years of the course will be classified, and those wishing to graduate at this point may supplicate for a BA.

Students wishing to take the four-year course should register to do so at the beginning of their third year. They will take Part C in their fourth year, awarded a separate classification and allowed to supplicate for an MMath.

The Schedule of papers

Altogether, these papers will include 1 short question and 1 longer question for each 8 lecture course; 2 short questions and 2 longer questions for each 16 lecture course; 3 short questions and 3 longer questions for each 24 lecture course.

Paper AC1 Algebra, Analysis and Differential Equations This paper will contain questions set on the CORE material, and will contain 9 short questions, attracting 10 marks each, all of which should be answered; 10 further marks will be available for presentation.

Paper AC2 Algebra, Analysis and Differential Equations This paper will contain questions set on the CORE material, and will contain 9 longer questions, attracting 25 marks each. At most 5 answers may be submitted and the best 4 will be counted.

Paper AO1 Options This paper will contain questions set on the OPTIONAL material, and will contain 19 short questions, attracting 10 marks each, of which at most 10 should be answered with the best 9 answers being counted.

Paper AO2 Options This paper will contain questions set on the OPTIONAL material, and will contain 19 longer questions, attracting 25 marks each. At most 5 answers may be submitted and the best 4 will be counted.

Syllabus and Synopses The **syllabus** details in this booklet are those referred to in the *Examination Regulations* and have been approved by the Subfaculty Teaching Committee for examination in Trinity Term 2006.

The **synopses** in this booklet give some additional detail, and show how the material is split between the different lecture courses. They also include details of recommended reading.

1.2 Honour School of Mathematics & Philosophy

See the current edition of the *Examination Regulations* for the full regulations governing these examinations. For the Schedule of Mathematics Papers for Part A see the Supplement to the Undergraduate Handbook for the Honour School of Mathematics & Philosophy:

http://www.maths.ox.ac.uk/current-students/undergraduates/handbooks-synopses/mathsphil.shtml.

1.3 Honour School of Mathematics & Statistics

See the current edition of the *Examination Regulations* for the full regulations governing these examinations, and the details published by the Statistics Department:

http://www.stats.ox.ac.uk/courses.htm.

Papers AC1 and AC2 under the schedule of papers here are taken by candidates in Mathematics & Statistics.

1.4 Honour Schools of Computer Science and Mathematics & Computer Science

See the current edition of the *Examination Regulations* for the full regulations governing these examinations, and the details published in a handbook by the Computing Laboratory:

http://web.comlab.ox.ac.uk/oucl/courses/synopses.

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2 CORE MATERIAL

2.1 Syllabus

2.1.1 Algebra

Vector spaces over an arbitrary field, subspaces, direct sums; projection maps and their characterisation as idempotent operators.

Dual spaces of finite-dimensional spaces; annihilators; the natural isomorphism between a space and its second dual; dual transformations and their matrix representation with respect to dual bases.

Some theory of a single linear transformation on a finite-dimensional space: characteristic polynomial, minimal polynomial, Primary Decomposition Theorem, the Cayley–Hamilton theorem; diagonalisability; triangular form.

Real and complex inner product spaces. Orthogonal complements, orthonormal sets; the Gram–Schmidt process. Bessel's inequality; the Cauchy–Schwarz inequality.

The adjoint of a linear transformation on a finite-dimensional inner product space to itself. Eigenvalues and diagonalisability of self-adjoint linear transformations.

Commutative rings with unity, integral domains, fields; units, irreducible elements, primes.

Ideals and quotient rings; isomorphism theorems. The Chinese Remainder Theorem [classical case of $\mathbb{Z}$ only].

Maximal ideals and their quotient rings.

Euclidean rings and their properties: polynomial rings as examples, theorem that their ideals are principal; theorem that their irreducible elements are prime; uniqueness of factorisation.

The Gaussian integers.

2.1.2 Analysis

The topology of Euclidean space and its subsets, particularly $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$: open sets, closed sets, subspace topology; continuous functions and their characterisation in terms of pre-images of open or closed sets; connected sets, path-connected sets; compact sets, Heine-Borel Theorem.

The algebra and geometry of the complex plane. Complex differentiation. Holomorphic functions. Cauchy-Riemann equations (including z, $\bar{z}$ version). Real and imaginary parts of a holomorphic function are harmonic.

Path integration. Fundamental Theorem of Calculus in the path integral/holomorphic function setting. Power series and differentiation of power series. Exponential function, logarithm function, fractional powers - examples of multifunctions.

Cauchy's Theorem (proof excluded). Cauchy's Integral formulae. Taylor expansion. Liouville's Theorem. Identity Theorem. Morera's Theorem. Laurent's expansion. Classification of singularities. Calculation of principal parts and residues. Residue theorem. Evaluation of integrals by the method of residues (straight forward examples only but to include use of Jordan's lemma and simple poles on contour of integration).

Conformal mapping: Möbius functions, exponential functions, fractional powers; mapping regions (not Christoffel transformations or Jowkowski's transformation).

2.1.3 Differential Equations

Picard's theorem for first-order scalar ODEs with proof. Statement, without proof, of extension to systems. Examples with blow-up and non-uniqueness.

Second-order ODEs: variation of parameters, Wronskian and Green's function.

Phase planes, critical points, Poincaré-Bendixson criterion. Examples including conservative nonlinear oscillators, van der Pol's equation and Lotka-Volterra equations. Stability of periodic solutions.

Characteristic methods for first-order quasilinear PDEs. Examples from conservation laws. Multivalued solutions and shocks.

Classification of second-order linear PDEs. Ideas of uniqueness and well-posedness for Laplace, wave and heat equations. Illustration of suitable boundary conditions by example. Multi-dimensional Laplacian operator giving rise to Bessel's and Legendre's equations.

Theory of Fourier and Laplace transforms, inversion, convolution. Inversion of some standard Fourier and Laplace transforms via contour integration. Use of Fourier transform in solving Laplace's equation and the heat equation. Use of Laplace transform in solving the heat equation.

2.2 Synopses of Lectures

2.2.1 Algebra — Dr Neumann — 24 lectures MT

Rings and arithmetic [8 lectures, Michaelmas Term]

Aims and Objectives This half-course introduces the student to some classic ring theory which is basic for other parts of abstract algebra, for linear algebra and for those parts of number theory that lead ultimately to applications in cryptography. The first-year algebra course contains a treatment of the Euclidean Algorithm in its classical forms for integers and for polynomial rings over a field. Here the idea is developed *in abstracto*. The Gaussian integers, which have applications to many questions of elementary number theory, give an important and interesting (and entertaining) illustration of the theory.

Synopsis Commutative rings with unity, integral domains, fields; examples including polynomial rings and subrings of $\mathbb{R}$ and $\mathbb{C}$.

Ideals and quotient rings; isomorphism theorems. Examples: integers modulo a natural number; the Chinese Remainder Theorem; the quotient ring by a maximal ideal is a field.

Arithmetic in integral domains. Units, associates, irreducible elements, primes. Euclidean rings and their properties: Division Algorithm, Euclidean Algorithm; $\mathbb{Z}$ and $F[x]$ as prototypes. Properties of euclidean rings: their ideals are principal; their irreducible elements are prime; factorisation is unique.

Examples for applications: Gauss's Lemma and factorisation in $\mathbb{Q}[x]$; factorisation of Gaussian integers.

Reading PETER J CAMERON, *Introduction to Algebra*, OUP 1998, ISBN 0-19-850194-3. Chapter 2.

Alternative and further reading:

JOSEPH J ROTMAN, *A First Course in Abstract Algebra*, (Second edition), Prentice Hall, 2000, ISBN 0-13-011584-3. Chapters 1, 3.

I N HERSTEIN, *Topics in Algebra* (Second edition) Wiley 1975, ISBN 0-471-02371-X. Chapter 3. [Harder than some, but an excellent classic. Widely available in Oxford libraries; still in print.]

P M COHN, *Classic Algebra*, Wiley 2000, ISBN 0-471-87732-8. Various sections. [This is the third edition of his book previously called *Algebra I*.]

DAVID SHARPE, *Rings and Factorization*, CUP 1987, ISBN 0-521-33718-6. [An excellent little book, now sadly out of print; available in some libraries, though.]

There are very many other such books on abstract algebra in Oxford libraries.

Further Linear Algebra [16 lectures in Michaelmas Term]

Aims and Objectives The core of linear algebra comprises the theory of linear equations in many variables, the theory of matrices and determinants, and the theory of vector spaces and linear transformations. All these topics were introduced in the Moderations course. Here they are developed further to provide the tools for applications in geometry, modern mechanics and theoretical physics, probability and statistics, functional analysis and, of course, algebra and number theory. Our aim is to provide a thorough treatment of some classical theory that describes the behaviour of linear transformations on a finite-dimensional vector space to itself, both in the purely algebraic setting and in the situation where the vector space carries a metric deriving from an inner product.

Synopsis Fields; $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{F}_p$ as examples. Vector spaces over an arbitrary field, subspaces, direct sums; projection maps and their characterisation as idempotent operators.

Dual spaces of finite-dimensional spaces; annihilators; the natural isomorphism between a finite-dimensional space and its second dual; dual transformations and their matrix representation with respect to dual bases.

Some theory of a single linear transformation on a finite-dimensional space: characteristic polynomial, minimal polynomial, Primary Decomposition Theorem, the Cayley–Hamilton theorem (economically); diagonalisability; triangular form.

Real and complex inner product spaces: examples, including function spaces [but excluding completeness and L^2]. Orthogonal complements, orthonormal sets; the Gram–Schmidt process. Bessel's inequality; the Cauchy–Schwarz inequality.

Some theory of a single linear transformation on a finite-dimensional inner product space: the adjoint; eigenvalues and diagonalisability of a self-adjoint linear transformation.

Reading RICHARD KAYE and ROBERT WILSON, *Linear Algebra*, OUP 1998, ISBN 0-19-850237-0. Chapters 2–13. [Chapters 6, 7 are not entirely relevant to our syllabus, but are interesting.]

Alternative and further reading:

PAUL R HALMOS *Finite-dimensional Vector Spaces*, (Reprint 1993 of the 1956 second edition), Springer Verlag ISBN 3-540-90093-4. §§1–15, 18, 32–51, 54–56, 59–67, 73, 74, 79. [Now over 50 years old, this idiosyncratic book is somewhat dated but it is a great classic, and well worth reading.]

P M COHN, *Classic Algebra*, Wiley 2000, ISBN 0-471-87732-8. §§4.1–4.7, 7.1–7.3, 8.1–8.5. [Third edition of his earlier *Algebra, I*. Does not cover the whole syllabus—but does have relevant material from other parts of algebra.]

SEYMOUR LIPSCHUTZ AND MARC LIPSON, *Schaum's Outline of Linear Algebra* (3rd edition, McGraw Hill 2000), ISBN 0-07-136200-2. [Many worked examples.]

C W CURTIS, *Linear Algebra—an Introductory Approach*, Springer (4th edition), reprinted 1994

D T FINKBEINER, *Elements of Linear Algebra*, Freeman, 1972 [Out of print, but available in many libraries]

2.2.2 Analysis — Dr Luke — 24 lectures MT

Aims and Objectives The theory of functions of a complex variable is a rewarding branch of mathematics to study at the undergraduate level with a good balance between general theory and examples. It occupies a central position in mathematics with links to analysis, algebra, number theory, potential theory, geometry, topology, and generates a number of powerful techniques (for example, evaluation of integrals) with applications in many aspects of both pure and applied mathematics, and other disciplines, particularly the physical sciences.

In these lectures we begin by introducing students to the language of topology before using it in the exposition of the theory of (holomorphic) functions of a complex variable. The central aim of the lectures is to present Cauchy's theorem and its consequences, particularly series expansions of holomorphic functions, the calculus of residues and its applications.

The course concludes with an account of the conformal properties of holomorphic functions and applications to mapping regions.

Synopsis (1-4) Topology of Euclidean space and its subsets, particularly $\mathbb{R}$, $\mathbb{R}^2$, $\mathbb{R}^3$. Open sets, closed sets, subspace topology; continuous functions and their characterisation in terms of preimages of open or closed sets; connected sets, path-connected sets; compact sets, Heine-Borel Theorem (covered in Chapter 3 of Apostol).

(5-7) Review of algebra and geometry of the complex plane. Complex differentiation. Holomorphic functions. Cauchy-Riemann equations. Real and imaginary parts of a holomorphic function are harmonic.

(8-11) Path integration. Power series and differentiation of power series. Exponential function and logarithm function. Fractional powers - examples of multifunctions.

(12-13) Cauchy's Theorem. (Sketch of proof only -students referred to various texts for proof) Fundamental Theorem of Calculus in the path integral/holomorphic situation.

(14-16) Cauchy's Integral formulae. Taylor expansion. Liouville's Theorem. Identity Theorem. Morera's Theorem

(17-18) Laurent's expansion. Classification of singularities. Calculation of principal parts, particularly residues.

(19-21) Residue theorem. Evaluation of integrals by the method of residues (straight forward examples only but to include use of Jordan's lemma and simple poles on contour of integration).

(22-23) Conformal mapping: Möbius functions, exponential functions, fractional powers; mapping regions (not Christoffel transformations or Jowkowski's transformation).

(24) Summary and Conclusion.

Reading Main texts

APOSTOL, *Mathematical Analysis*, Addison–Wesley (Chapter 3 for the topology).

PRIESTLEY, *Introduction to Complex Analysis*, (second edition, 2003), Oxford Science Publications.

JEROLD E MARSDEN, MICHAEL J HOFFMAN, *Basic Complex Analysis*, W.H. Freeman.

Further Reading THEODORE GAMELIN,*Complex Analysis*, Springer (2000).

REINHOLD REMMERT, *Theory of Complex Functions*, Springer (1989) (Graduate Texts in Mathematics 122)

MARK J ABLOWITZ, ATHANASSIOS S. FOCAS, *Complex Variables, Introduction and Applications*, Cambridge Texts in Applied Mathematics.

2.2.3 Differential Equations — Prof Tod — 24 lectures MT

Aims and Objectives The aim of this course is to introduce all students reading mathematics to the basic theory of ordinary and partial differential equations. On completion of the course, students will understand the importance of existence and uniqueness and will be aware that explicit analytic solutions are the exception rather than the rule. They will acquire a toolbox of methods for solving linear equations and for understanding the solutions of nonlinear equations.

The course will be example-led and will concentrate on equations that arise in practice rather than those constructed to illustrate a mathematical theory. The emphasis will be on solving equations and understanding the possible behaviours of solutions, and the analysis will be developed as a means to this end.

The course will furnish undergraduates with the necessary skills to pursue any of the applied options in the third year and will also form the foundation for a deeper and more rigorous course in partial differential equations.

Synopsis (1–4) Picard's theorem for $\mathrm{d}y/\mathrm{d}x = f(x, y)$ with proof. Extension to systems stated but not proved. Examples with blow-up and non-uniqueness.
(Boyce & DiPrima section 2.12. Kreyszig section 1.9. Collins section 2.1.)

(5–7) Second-order ODEs: variation of parameters, Wronskian and Green's function.
(Boyce & DiPrima sections 3.1–3.5, 3.6, 3.6.2. Hildebrand chapter 3. Kreyszig sections 2.1, 2.7–2.10. Collins chapters 3, 4.)

(8–11) Phase planes, critical points, Poincaré-Bendixson criterion. Examples including conservative nonlinear oscillators, van der Pol's equation and Lotka-Volterra equations. Stability of periodic solutions.
(Boyce & DiPrima sections 9.1–9.4. Kreyszig sections 3.3–3.5.)

(12–14) Characteristic methods for first-order quasilinear PDEs (using parameterisation). Examples from conservation laws. Multivalued solutions and shocks. *Charpit's method and artificial examples excluded.*
(Carrier & Pearson chapters 6, 13. Ockendon *et al.* chapter 1. Collins chapter 5.)

(15–18) Classification of second-order linear PDEs. Ideas of uniqueness and well-posedness for Laplace, Wave and Heat equations. Revision of separation of variables from Mods and illustration of suitable boundary conditions by example. Multi-dimensional Laplacian operator giving rise to Bessel's and Legendre's equations.
(Carrier & Pearson chapters 1, 3, 4, 5, 7. Strauss chapter 1. Kreyszig sections 11.7–11.11.)

(19–24) Theory of Fourier and Laplace transforms, inversion, convolution. Inversion of some standard Fourier and Laplace transforms via contour integration. Use of Fourier transform in solving Laplace's equation and the heat equation. Use of Laplace transform in solving the heat equation.
(Carrier & Pearson chapters 2, 15. Kreyszig chapter 5, sections 10.8–10.11, 11.6. Priestley chapter 9.)

Reading W E BOYCE & R C DIPRIMA, *Elementary Differential Equations and Boundary Value Problems, 7th edition*, Wiley (2000).

ERWIN KREYSZIG, *Advanced Engineering Mathematics, 8th Edition*, Wiley (1999).

F B HILDEBRAND, *Methods of Applied Mathematics*, Dover (1992).

W A STRAUSS, *Partial Differential Equations: an Introduction*, Wiley (1992).

G F CARRIER & C E PEARSON, *Partial Differential Equations — Theory and Technique*, Academic (1988).

H A PRIESTLEY, *Introduction to Complex Analysis*, second edition 2003, Oxford (1990).

J OCKENDON, S HOWISON, A LACEY & A MOVCHAN, *Applied Partial Differential Equations*, Oxford (1999). [More advanced]

Additional Reading P J COLLINS, *D*ifferential and Integral Equations, O.U.P., Chapters 1-7, 14,15.

3 OPTIONS

3.1 Syllabus

3.1.1 Groups in Action

Groups: subgroups, normal subgroups and quotient groups; elementary results concerning symmetric and alternating groups; important examples of groups. Actions of groups on sets; examples, including coset spaces, groups acting on themselves by conjugation, the Möbius group acting on the Riemann sphere. Permutation representations; the permutation representation associated with a coset space; examples. Orbits, transitivity, stabilisers, isomorphism of a transitive space with a coset space, examples. Symmetry groups of geometric objects including regular polyhedra. Combinatorial applications: "Not Burnside's Lemma" and its use in enumeration problems.

3.1.2 Introduction to Fields

Fields, subfields, finite extensions; examples. Degree of an extension, the Tower Theorem. Simple algebraic extensions; splitting fields, uniqueness (proof not to be examined); examples. Characteristic of a field. Finite fields: existence; uniqueness (proof not to be examined). Subfields. The multiplicative group of a finite field. The discrete logarithm.

3.1.3 Number Theory

The ring of integers; congruences; rings of integers modulo n; the Chinese Remainder Theorem. Wilson's Theorem; Fermat's Little Theorem for prime modulus. Euler's phi-function; Euler's generalisation of Fermat's Little Theorem to arbitrary modulus. Quadatic residues modulo primes. Quadratic reciprocity. Factorisation of large integers; basic version of the RSA encryption method.

3.1.4 Integration

The Lebesgue integral on the real line, null sets, reconciliation with the integral introduced in Moderations. [Details of technical results on the construction of the integral are not examinable.] Properties of the Lebesgue integral. Simple examples of non-integrable functions.

The Monotone Convergence Theorem [proof not examinable] and the deduction of the Dominated Convergence Theorem. Integrability of x^a, e^{-x} and e^{-x^2} over suitable intervals; simple form of comparison theorem for integrability. Term-by-term integration of a series including Lebesgue's Series Theorem.

The Dominated Convergence Theorem with a continuous parameter; examples of continuity and differentiation of functions defined by integrals.

Simple examples of improper integrals. Integrals of complex-valued functions. Measurable functions (defined as limits a.e. of sequences of step functions); comparison theorem for integrability. Measurable sets and Lebesgue measure, including countable additivity.

Outline development of the Lebesgue integral on $\mathbb{R}^2$. The theorems of Fubini and Tonelli (proof of technical lemma on null sets is not examinable). Convolution of integrable functions. Simple form of change of variables theorem (no proof). Examples.

L^p-spaces as vector spaces. [Completeness and proofs of Hölder's inequality and Minkowski's inequality for general p are not examinable.]

3.1.5 Topology

Metric spaces. Examples to include metrics derived from a norm on a real vector space, particularly l^1, l^2, l^∞ norms on $\mathbb{R}^n$, the *sup* norm on the bounded real-valued functions on a set, and on the bounded continuous real-valued functions on a metric space. Continuous functions (ϵ, δ definition). Uniformly continuous functions. Open balls, open sets, accumulation points of a set. Completeness (but not completion). Contraction mapping theorem. Completeness of the space of bounded real-valued functions on a set, equipped with the *sup* norm, and the completeness of the space of bounded continuous real-valued functions on a metric space, equipped with the *sup* metric.

Axiomatic definition of an abstract topological space in terms of open sets. Continuous functions, homeomorphisms. Closed sets. Accumulation points of sets. Closure of a set ($\bar{A} = A$ together with its accumulation points). Continuity if $f(\bar{A}) \subseteq \overline{f(A)}$. Examples to include metric spaces (definition of topological equivalence of metric spaces), discrete and indiscrete topologies, subspace topology, cofinite topology, quotient topology. Base of a topology. Product topology on a product of two spaces and continuity of projections. Hausdorff topology.

Connected spaces: closure of a connected space is connected, union of connected sets is connected if there is a non-empty intersection, continuous image of a connected space is connected. Path-connectedness implies connectedness. Connected open subset of a normed vector space is path-connected.

Compact sets, closed subset of a compact set is compact, compact subset of a Hausdorff space is closed. Heine-Borel Theorem in $\mathbb{R}^n$. Product of two compact spaces is compact. A continuous bijection from a compact space to a Hausdorff space is a homeomorphism. Equivalence of sequential compactness and abstract compactness in metric spaces.

Further discussion of quotient spaces: simple classical geometric spaces such as the torus and Klein bottle.

3.1.6 Multivariable Calculus

Definition of a derivative of a function from $\mathbb{R}^m$ to $\mathbb{R}^n$; examples; elementary properties; partial derivatives; the chain rule; the gradient of a function from $\mathbb{R}^m$ to $\mathbb{R}$; Jacobian. Continuous partial derivatives imply differentiability. Higher order derivatives; symmetry of multiple partial derivatives.

The inverse function theorem (proof non-examinable). The implicit function theorem (statement only).

The definition of a submanifold of $\mathbb{R}^m$, its tangent space at a point. Examples, defined parametrically and implicitly, including curves and surfaces in $\mathbb{R}^3$, and $SO(n)$.

Lagrange multipliers.

3.1.7 Calculus of Variations

The basic variational problem and Euler's equation. Examples, including axisymmetric soap films. Extension to several dependent variables. Hamilton's principle for free particles and particles subject to holonomic constraints. Equivalence with Newton's second law. Geodesics on surfaces. Extension to several independent variables. Examples including Laplace's equation. Lagrange multipliers and variations subject to constraint. The Rayleigh-Ritz method and eigenvalue problems for Sturm-Liouville equations.

3.1.8 Classical Mechanics

Angular momentum of a system of particles about a fixed point and about the centre of mass. The description of the motion of a rigid body with one fixed point in terms of a time-dependent rotation matrix. Definition of angular velocity. Moments of inertia, kinetic energy, and angular momentum of a rigid body with axial symmetry. Lagrangian equations of motion. Gyroscopes and the classical integrable cases of rigid body motion. Oscillations near equilibrium; normal frequencies, normal modes.

3.1.9 Electromagnetism

Charged particles and their currents; conservation of charge and the continuity equation; Coulomb's Law and definition of $\underline{E}$; the electrostatic potential; Gauss' Law; boundary conditions at infinity and on surfaces.

Electric currents, Biot-Savart law and the definition of $\underline{B}$, Ampère's Law; absence of magnetic charges; the Lorentz force; the magnetic potential.

Time-dependent electromagnetic fields and Maxwell's equations; continuity equation (as consistency of Maxwell's equations); Poynting Vector and Poynting Theorem; potentials and gauge invariance; electromagnetic plane-wave solutions; polarisation and reflection.

3.1.10 Fluid Dynamics and Waves

Incompressible flow. Convective derivative, streamlines and particle paths. Euler's equations of motion for an inviscid fluid. Bernoulli's theorem. Vorticity, circulation and Kelvin's theorem.

Irrotational incompressible flow; velocity potential. Two-dimensional flow, stream function and complex potential. Line sources and vortices. Method of images, circle theorem and Blasius's theorem.

Uniform flow past a circular cylinder. Circulation, lift. Use of conformal mapping to determine flow past a flat wing. Water waves, including effects of finite depth and surface tension. Dispersion, simple introduction to group velocity. The vorticity equation and vortex motion.

3.1.11 Probability

Random variables and their distribution; joint distribution, conditional distribution; functions of one or more random variables. Generating functions and applications. Characteristic functions, definition only. Statements of the continuity and uniqueness theorems for moment generating functions. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable). Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

3.1.12 Statistics

Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.

3.1.13 Numerical Analysis

Lagrange interpolation, Newton-Cotes quadrature, Gaussian elimination and LU factorization, QR factorization. Eigenvalues: Gershgorin's theorem, symmetric QR algorithm. Best approximation in inner product spaces, least squares, orthogonal polynomials. Piecewise polynomials, splines, Richardson Extrapolation.

3.2 Synopses of Lectures

3.2.1 Groups in Action — Dr M Collins — 8 lectures HT

Aims and Objectives The beginnings of the theory of groups were presented in the Moderations course in algebra. Here we develop that theory, aiming particularly to show how it is used in the measurement of symmetry, what symmetry is and how to show it may be measured is relatively easy to understand in geometrical contexts. But it is important in most other parts of mathematics, in much of physics and chemistry, and in many other areas. Our aim in this half unit is to introduce the theory of actions of groups and show how it may be applied in the measurement of symmetry.

Synopsis Reminders about groups, subgroups, normal subgroups and quotient groups; reminders about the symmetric and alternating groups; important examples of groups. [1 lecture]

Actions of groups on sets; examples including coset spaces, groups acting on themselves by conjugation, the Möbius group acting on the Riemann sphere. [1 lecture]

Permutation representations; the permutation representation associated with a coset space; examples. [2 lectures]

Orbits, transitivity, stabilisers, isomorphism of a transitive space with a coset space, examples. [2 lectures]

Symmetry groups of geometric objects including regular polyhedra. Combinatorial applications: "Not Burnside's Lemma" and its use in enumeration problems. [2 lectures]

Reading PETER M NEUMANN, G A STOY, E C THOMPSON, *Groups and Geometry*, (OUP 1994, reprinted 2002), ISBN 0-19-853451-5. Chapters 1-9, 15.

Further Reading

GEOFF SMITH, OLGA TABACHNIKOVA, *Topics in Groups Theory* (Springer Undergraduate Mathematics Series, 2002) ISBN 1-85233-2. Chapter 3.

M A ARMSTRONG, *Groups and Symmetry*, (Springer 1988), ISBN 0-387-96675-7. Chapters 1-19.

JOSEPH J ROTMAN, *A First Course in Algebra*, (Second Edition, Prentice Hall, 2000). Chapter 2

3.2.2 Introduction to Fields — Dr M Collins — 8 lectures HT

Aims and Objectives Informally, finite fields are generalisations of systems of real numbers such as the rational or the real numbers— systems in which the usual rules of arithmetic (including those for division) apply. Formally, fields are commutative rings with unity in which division by non-zero elements is always possible. It is a remarkable fact that the finite fields may be completely classified. Furthermore, they have classical applications in number theory, algebra, geometry, combinatorics, and coding theory, and they have newer applications in other areas. The aim of this course is to show how their structure may be elucidated, and to present the main theorems about them that lead to their various applications.

Synopsis Fields, subfields, finite extensions; examples. Degree of an extension, the Tower Theorem. [2 lectures]

Simple algebraic extensions; splitting fields, uniqueness (proof only sketched); examples. [2 lectures]

Characteristic of a field. Finite fields: existence; uniqueness (proof only sketched). Subfields. The multiplicative group of a finite field. The discrete logarithm. [4 lectures]

Reading JOSEPH J ROTMAN, *A First Course in Abstract Algebra*, (Second Edition, Prentice Hall, 2000), ISBN 0-13-011584-3. Chapters 1,3.

DOMINIC WELSH, *Codes and Cryptography*, Oxford University Press 1988, ISBN 0-19853-287-3. Chapter 10.

Further Reading

PETER J CAMERON, *Introduction to Algebra*, Oxford Univeristy Press 1998, ISBN 0-19-850194-3 Parts of 2.4, 7.3.

I N HERSTEIN, *Topics in Algebra*, (Wiley 1975). ISBN 0-471-02371-X 5.1, 5.3, 7.1. [Harder than some, but an excellent classic. Widely available in Oxford libraries; still in print.]

P M COHN, *Classic Algebra*, (Wiley 2000), ISBN 0-471-87732-8, parts of Chapter 6. [This is the third edition of his book on abstract algebra, in Oxford libraries.]

There are many other such books on abstract algebra in Oxford libraries

3.2.3 Number Theory — Prof Heath-Brown — 8 lectures TT

Aims and Objectives Number theory is one of the oldest parts of mathematics. For well over two thousand years it has attracted professional and amateur mathematicians alike. Although notoriously ‘pure’ it has turned out to have more and more applications as new subjects and new technologies have developed. Our aim in this course is to introduce students to some classical and important basic ideas of the subject.

Synopsis The ring of integers; congruences; rings of integers modulo u; the Chinese Remainder Theorem. [2 lectures]

Wilson's Theorem; Fermat's Little Theorem for prime modulus; Euler's generalization of Fermat's Little Theorem to arbitrary modulus; primitive roots.[2 lectures]

Quadratic residues modulo primes. Quadratic reciprocity. [2 lectures]

Factorisation of large integers; basic version of the RSA encryption method. [2 lectures]

Reading ALAN BAKER, *A Concise Introduction to the Theory of Numbers*, Cambridge University Press ISBN: 0521286549 Chapters 1,3,4

DAVID BURTON, *Elementary Number Theory*, (McGraw-Hill, 2001).

DOMINIC WELSH, *Codes and Cryptography*, Oxford University Press 1988, ISBN 0-19853-287-3. Chapter 11

3.2.4 Integration — Dr Vincent-Smith — 16 lectures HT

Aims and Objectives A naive integral was introduced in the Moderations syllabus. For more advanced applications where functions are not continuous and limiting processes are involved, a more advanced integral is needed. The aim of this course is to introduce students to the Lebesgue integral, its properties and applications. This facilitates the understanding of integration on the more abstract spaces that underpin probability theory.

This option is good preparation for later units on mathematical analysis, functional analysis, differential equations, applied analysis, probability theory and many applied mathematics topics.

Synopsis The Lebesgue integral on the real line, null sets, reconciliation with the integral introduced in Moderations. [Details of technical results on the construction of the integral are not examinable.] Properties of the Lebesgue integral. Simple examples of non-integrable functions.

The Monotone Convergence Theorem [proof not examinable] and the deduction of the Dominated Convergence Theorem. Integrability of x^a, e^{-x} and e^{-x^2} over suitable intervals; simple form of comparison therem for integrability. Term-by-term integration of a series including Lebesgue's Series Theorem.

The Dominated Convergence Theorem with a continuous parameter; examples of continuity and differentiation of functions defined by integrals.

Simple examples of improper integrals. Integrals of complex-valued functions. Measurable functions (defined as limits a.e. of sequences of step functions); comparison theorem for integrability. Measurable sets and Lebesgue measure, including countable additivity.

Outline development of the Lebesgue integral on $\mathbb{R}^2$. The theorems of Fubini and Tonelli (proof of technical lemma on null sets is not examinable). Convolution of integrable functions. Simple form of change of variables theorem (no proof). Examples.

L^p-spaces as vector spaces. [Proofs of Hölder's inequality and Minkowski's inequality for general p, and completeness are not examinable.]

Reading H PRIESTLEY, *Introduction to Integration*, Oxford Science Publications, 1997, Chapters 1-28.

3.2.5 Topology — Dr Sutherland — 16 lectures HT

Learning Outcomes The ideas, concepts and constructions in general topology arose from extending the notions of continuity and convergence on the real line to more general spaces. The first class of general spaces to be studied in this way were metric spaces, a class of spaces which includes many of the spaces used in analysis and geometry. Metric spaces have a distance function which allows the use of geometric intuition and gives them a concrete feel. They allow us to introduce much of the vocabulary used later and to understand the formulation of continuity which motivates the axioms in the definition of an abstract topological space.

The axiomatic formulation of a topology leads to topological proofs of simplicity and clarity often improving on those given for metric spaces using the metric and sequences. There are many examples of topological spaces which do not admit metrics and it is an indication of the naturality of the axioms that the theory has found so many applications in other branches of mathematics and spheres in which mathematical language is used.

The outcome of the course is that a student should understand and appreciate the central results of general topology and metric spaces, sufficient for the main applications in geometry, number theory, analysis and mathematical physics, for example.

Synopsis Metric spaces. Examples to include metrics derived from a norm on a real vector space, particularly l^1, l^2, l^∞ norms on $\mathbb{R}^n$, the *sup* norm on the bounded real-valued functions on a set, and on the bounded continuous real-valued functions on a metric space. Continuous functions (ϵ, δ definition). Uniformly continuous functions. Open balls, open sets, accumulation points of a set. Completeness (but not completion). Contraction mapping theorem. Completeness of the space of bounded real-valued functions on a set, equipped with the *sup* norm, and the completeness of the space of bounded continuous real-valued functions on a metric space, equipped with the *sup* metric. [3 lectures].

Axiomatic definition of an abstract topological space in terms of open sets. Continuous functions, homeomorphisms. Closed sets. Accumulation points of sets. Closure of a set ($\bar{A} = A$ together with its accumulation points). Continuity if $f(\bar{A}) \subseteq \overline{f(A)}$. Examples to include metric spaces (definition of topological equivalence of metric spaces), discrete and indiscrete topologies, subspace topology, cofinite topology, quotient topology. Base of a topology. Product topology on a product of two spaces and continuity of projections. Hausdorff topology. [5 lectures]

Connected spaces: closure of a connected space is connected, union of connected sets is connected if there is a non-empty intersection, continuous image of a connected space is connected. Path-connectedness implies connectedness. Connected open subset of a normed vector space is path-connected. [2 lectures]

Compact sets, closed subset of a compact set is compact, compact subset of a Hausdorff space is closed. Heine-Borel Theorem in $\mathbb{R}^n$. Product of two compact spaces is compact. A continuous bijection from a compact space to a Hausdorff space is a homeomorphism. Equivalence of sequential compactness and abstract compactness in metric spaces. [4 lectures]

Further discussion of quotient spaces explaining some simple classical geometric spaces such as the torus and Klein bottle. [2 lectures]

Reading W A SUTHERLAND, *Introduction to Metric and Topological Spaces*, OUP (1975). Chapters 2-6, 8, 9.1-9.4

Further Reading

B MENDELSON, *Introduction to Topology*, Allyn and Bacon (1975). (cheap paper back edition available).

G BUSKES, A VAN ROOIJ, *Topological Spaces*, Springer (1997).

J R MUNKRES *Topology, A First Course*, Prentice Hall (1974).

N BOURBAKI, *General Topology*, Springer (1998).

R ENGELKING, *General Topology*, Heldermann Verlag Berlin (1989) (for the last word, goes far beyond the syllabus).

3.2.6 Multivariable Calculus — Dr Day — 8 lectures TT

Aims and Objectves In this course, the notion of a derivative for a function $f\colon \mathbb{R}^m \to \mathbb{R}^n$ is introduced. Roughly speaking, this is an approximation of the function near each point in $\mathbb{R}^n$ by a linear transformation. This is a key concept which pervades much of mathematics, both pure and applied, but is particularly important in geometry.

Synopsis Definition of a derivative of a function from $\mathbb{R}^m$ to $\mathbb{R}^n$; examples; elementary properties; partial derivatives; the chain rule; the gradient of a function from $\mathbb{R}^m$ to $\mathbb{R}$; Jacobian. Continuous partial derivatives imply differentiability. Higher order derivatives; symmetry of multiple partial derivatives. [3 lectures]

The inverse function theorem (proof non-examinable), via the contraction mapping theorem. The implicit function theorem (statement only) [2 lectures]

The definition of a submanifold of $\mathbb{R}^m$. Its tangent space at a point. Examples, defined parametrically and implicitly, including curves and surfaces in $\mathbb{R}^3$, and $SO(n)$. [2 lectures]

Lagrange multipliers. [1 lecture]

Reading THEODORE SHIFRIN *Multivariable Mathematics*, Wiley (2005). Chapters 3-6.

T M APOSTOL, *Mathematical Analysis: Modern Approach to Advanced Calculus (World Students)* Addison Wesley (1975) Chapters 12 and 13.

S DINEEN, *Multivariate Calculus and Geometry*, (Springer 2001) Chapters 1-4.

J J DUISTERMAAT and J A C KOLK, *Multidimensional Real Analysis I, Differentiation*, Cambridge University Press (2004).

B D CRAVEN, *Functions of Several Variables* (Chapman & Hall 1981) ISBN: 0-412-23330-4 (out of print) Chapters 1-3.

Further reading

WILLIAM R WADE, *An Introduction to Analysis*, 2nd Edition, 2000, Prentice Hall, Chapter 11

C DIXON, *Advanced Calculus* (Wiley 1981) (out of print)

M P DO CARMO, *Differential Geometry of Curves and Surfaces.*

3.2.7 Calculus of Variations — Prof Maini — 8 lectures HT

Aims and Objectives The calculus of variations concerns problems in which one wishes to find the minima or extrema of some quantity over a system that has functional degrees of freedom. Many important problems arise in this way across pure and applied mathematics and physics. They range from the problem in geometry of finding the shape of a soap bubble, a surface that minimizes its surface area, to finding the configuration of a piece of elastic that minimises its energy. Perhaps most importantly, the principle of least action is now the standard way to formulate the laws of mechanics and basic physics.

In this course it is shown that such variational problems give rise to a system of differential equations, the Euler-Lagrange equations. Furthermore, the minimizing principle that underlies these equations leads to direct methods for analysing the solutions to these equations. These methods have far reaching applications and will help develop students technique.

Learning Outcomes Students will learn how to formulate variational problems and analyse them to deduce key properties of system behaviour.

Synopsis The basic variational problem and Euler's equation. Examples, including axisymmetric soap films. Extension to several dependent variables. Hamilton's principle for free particles and particles subject to holonomic constraints. Equivalence with Newton's second law. Geodesics on surfaces. Extension to several independent variables. Examples including Laplace's equation. Lagrange multipliers and variations subject to constraint. The Rayleigh-Ritz method and eigenvalue problems for Sturm-Liouville equations.

Reading ARFKEN WEBER, *Mathematical Methods for Physicists* (5th edition, Academic Press). Chapter 17

Further Reading N M J WOODHOUSE, *Introduction to Analytical Dynamics* (1987). Chapter 2 (in particular 2.6). (This is out of print, but still available in most College libraries.)

M LUNN, *A* First Course in Mechanics, OUP. Chapters 8.1, 8.2

P J COLLINS, *D*ifferential and Integral Equations, O.U.P., Chapters 11, 12.

3.2.8 Classical Mechanics — Dr Piotr Chrusciel — 8 lectures HT

Aims and Objectives The aim of the course is to extend the study of dynamics in the first year to a class of three dimensional motions, and to introduce some key classical ideas that also play an important role in modern physical theory, notably angular momentum and its connection with rotations. Some of the classical systems studied have more recently been seen as key examples in modern approaches to the theory of integrability. The course also covers the powerful application of the Lagrangian theory to the study of small oscillations near equilibrium.

Synopsis Angular momentum of a system of particles about a fixed point and about the centre of mass. The description of the motion of a rigid body with one fixed point in terms of a time-dependent rotation matrix. Definition of angular velocity. Moments of inertia, kinetic energy, and angular momentum of a rigid body with axial symmetry. Lagrangian equations of motion. Gyroscopes and the classical integrable cases of rigid body motion. Oscillations near equilibrium; normal frequencies, normal modes.

Reading N M J WOODHOUSE, *Introduction to Analytical Mechanics* (1987). Chapters 3 and 6. (This is out of print, but still available in most College libraries.)

Further reading M LUNN, *A First Course in Mechanics*, OUP. Chapters 6, 7.2, 7.3, 8.3 and 8.4

3.2.9 Electromagnetism — Prof Tod — 8 lectures TT

Aims and Objectives The phenomena of electricity and magnetism are familiar from school physics. In this course we first put these phenomena into a theoretical mathematical framework expressed in terms of vector calculus, as an application of grad, div and curl from moderations. Then we follow the nineteeth century physicist Maxwell in unifiying the two subjects into the single theory of electromagnetism, one of the cornerstones of modern physics. With this theory we are able to describe light as an electromagnetic phenomenon and to derive some of their elementary properties, notably polarisation and reflection.

Synopsis Charged particles and their currents; conservation of charge and the continuity equation; Coulomb's Law and definition of $\underline{E}$; the electrostatic potential; Gauss' Law; boundary conditions at infinity and on surfaces.

Electric currents, Biot-Savart law and the definition of $\underline{B}$, Ampère's Law; absence of magnetic charges; the Lorentz force; the magnetic potential.

Time-dependent electromagnetic fields and Maxwell's equations; continuity equation (as consistency of Maxwell's equations); Poynting Vector and Poynting Theorem; potentials and gauge invariance; electromagnetic plane-wave solutions; polarisation and reflection.

Reading W.J. DUFFIN, *Electricity and Magnetism*, McGraw-Hill (2001), Fourth Edition, Chapters 1-4, 7, 8, 13.

Alternative Reading:

N.M.J. WOODHOUSE, *Special Relativity, Lecture Notes in Physics*, Springer-Verlag (2002), Chapters 2,3.

R. FEYNMAN, *Lectures in Physics*, Vol. 2. *Electromagnetism*, Addison Wesley.

P. LORRAIN, D.R. CORSON, *Electromagnetism*, Freeman, Second Edition, Chapters 2, 3, 4.3-5, 8-11, 19, 20.

B.I. BLEANEY and B. BLEANEY, *Electricity and Magnetism*, OUP, Third Edition, Chapters 1.1-4, 2 (except 2.3),3.1-2, 4.1-2, 4.4, 5.1, 8.1-4.

Further reading

An advanced text on electromagnetism is

J.D. JACKSON, *Classical Electrodynamics*, John Wiley (1999).

3.2.10 Fluid Dynamics and Waves — Dr Acheson — 16 lectures HT

Aims and Objectives This course introduces students to the mathematical theory of inviscid fluids. The theory provides insight into physical phenomena such as flight, vortex motion, and water waves. The course also explains important concepts such as conservation laws and dispersive waves and, thus, serves as an introduction to the mathematical modelling of continuous media.

Synopsis Incompressible flow. Convective derivative, streamlines and particle paths. Euler's equations of motion for an inviscid fluid. Bernoulli's theorem. Vorticity, circulation and Kelvin's theorem.

Irrotational incompressible flow; velocity potential. Two-dimensional flow, stream function and complex potential. Line sources and vortices. Method of images, circle theorem and Blasius's theorem.

Uniform flow past a circular cylinder. Circulation, lift. Use of conformal mapping to determine flow past a flat wing. Water waves, including effects of finite depth and surface tension. Dispersion, simple introduction to group velocity. The vorticity equation and vortex motion.

Reading D J ACHESON *Elementary Fluid Dynamics*, OUP (1997) Chapters 1, 3.1-3.5, 4.1-4.8, 4.10-4.12, 5.1, 5.2, 5.6, 5.7.

3.2.11 Probability — Lecturer TBA — 16 lectures HT

Aims and Objectives The first half of the course takes further the probability theory that was developed in the first year. The aim is to build up a range of techniques that will be useful in dealing with mathematical models involving uncertainty. The second half of the course is concerned with Markov chains in discrete time and Poisson processes in one dimension, both with developing the relevant theory and giving examples of applications.

Synopsis Continuous random variables. Jointly continuous random variables, independence, conditioning, bivariate distributions, functions of one or more random variables. Moment generating functions and applications. Characteristic functions, definition only. Examples to include some of those which may have later applications in Statistics.

Basic ideas of what it means for a sequence of random variables to converge in probability, in distribution and in mean square. Chebychev and Markov inequalities. The weak law of large numbers and central limit theorem for independent identically distributed variables with a second moment. Statements of the continuity and uniqueness theorems for moment generating functions.

Discrete-time Markov chains: definition, transition matrix, n-step transition probabilities, communicating classes, absorption, irreducibility, calculation of hitting probabilities and mean hitting times, recurrence and transience. Invariant distributions, mean return time, positive recurrence, convergence to equilibrium (proof not examinable). Examples of applications in areas such as: genetics, branching processes, Markov chain Monte Carlo. Poisson processes in one dimension: exponential spacings, Poisson counts, thinning and superposition.

Reading G R GRIMMETT and D R STIRZAKER, *Probability and Random Processes*, 3rd edition, OUP (2001) Chapters 4, 6.1-6.5, 6.8

G R GRIMMETT and D R STIRZAKER, *One Thousand Exercises in Probability*, OUP (2001)

G R GRIMMETT and D J A WELSH, *Probability: An Introduction*, OUP (1986) Chapters 6, 7.4, 8, 11.1-11.3

J R NORRIS, *Markov Chains*, CUP (1997) Chapter 1

D R STIRZAKER, *Elementary Probability*, CUP 2nd Edition (2003) Chapters 7-9 excluding 9.9

3.2.12 Statistics — Lecturer TBA — 16 lectures HT

Aims and Objectives Building on the first year course, this course develops statistics for mathematicians, emphasising both its underlying mathematical structure and its application to the logical interpretation of scientific data. Advances in theoretical statistics are generally driven by the need to analyse new and interesting data which come from all walks of life.

Synopsis Estimation: observed and expected information, statement of large sample properties of maximum likelihood estimators in the regular case, methods for calculating maximum likelihood estimates, large sample distribution of sample estimators using the delta method.

Hypothesis testing: simple and composite hypotheses, size, power and p-values, Neyman-Pearson lemma, distribution theory for testing means and variances in the normal model, generalized likelihood ratio, statement of its large sample distribution under the null hypothesis, analysis of count data.

Confidence intervals: exact intervals, approximate intervals using large sample theory, relationship to hypothesis testing.

Regression: correlation, least squares and maximum likelihood estimation, use of matrices, distribution theory for the normal model, hypothesis tests and confidence intervals for linear regression problems, examining assumptions by plotting residuals.

Examples: statistical techniques will be illustrated with relevant datasets in the lectures.

Reading F DALY, D J HAND, M C JONES, A D LUNN and K J McCONWAY, *Elements of Statistics*, Addison Wesley (1995) Chapters 7-10 (and Chapters 1-6 for background)

J A RICE, *Mathematical Statistics and Data Analysis*, 2nd edition, Wadsworth (1995) Sections 8.5, 8.6, 9.1-9.7, 9.9, 10.3-10.6, 11.2, 11.3, 12.2.1, 13.3, 13.4.

Further Reading G CASELLA and R L BERGER, *Statistical Inference*, 2nd edition, Wadsworth (2001)

3.2.13 Numerical Analysis — Prof Gould — 16 lectures HT

Aims and Objectives Scientific computing pervades our lives: modern buildings and structures are designed using it, medical images are reconstructed for doctors using it, the cars and planes we travel on are designed with it, the pricing of “instruments” in the financial market are done using it, tomorrows weather is predicted with it.

The derivation and study of the core, underpinning algorithm for this vast range of applications defines the subject of Numerical Analysis. This course gives an introduction to that subject.

Through studying the material of this course students should gain an understanding of numerical methods, their derivation, analysis and applicability. They should be able to solve certain mathematically posed problems using numerical algorithms.

This course is designed to introduce numerical methods - i.e. techniques which lead to the (approximate) solution of mathematical problems which are usually implemented on computers. The course covers derivation of useful methods and analysis of their accuracy and applicability.

The course begins with a study of methods and errors associated with computation of functions which are described by data values (interpolation or data fitting). Following this we turn to numerical methods of linear algebra, which form the basis of a large part of computational mathematics, science, and engineering. Key ideas here include algorithms for linear equations, least squares, and eigenvalues built on LU and QR matrix factorizations. The course will also include the simple and computationally convenient approximation of curves: this includes the use of splines to provide a smooth representation of complicated curves, such as arise in computer aided design. Use of such representations leads to approximate methods of integration. Techniques for improving accuracy through extrapolation will also be described.

The course requires elementary knowledge of functions and calculus and of linear algebra.

Although there are no assessed practicals for this course, the classwork will involve a mix of written work and Matlab programming. No previous knowledge of Matlab is required. Specifically, Numerical Analysis has 16 lectures, no practicals, and 7 classes per term. There will be some simple use of Matlab – some laboratory time and demonstrations will be scheduled to introduce this.

Synopsis Lagrange interpolation [1 lecture],

Newton-Cotes quadrature [2 lectures],

Gaussian elimination and LU factorization [2 lectures],

QR factorization [1 lecture],

Eigenvalues: Gershgorins theorem, symmetric QR algorithm [3 lectures],

Best approximation in inner product spaces, least squares, orthogonal polynomials [4 lectures],

Piecewise polynomials, splines [2 lectures],

Richardson Extrapolation [1 lecture].

Reading The main book for this course is:

E SÜLI and D F MAYERS, *An Introduction to Numerical Analysis*, CUP, 2003, of which the relevant chapters are: 6, 7, 2, 5, 9, 11.

You can also find the material for this course in many introductory books on Numerical

Analysis such as,

A QUARTERONI, R SACCO and F SALERI, *Numerical Mathematics*, Springer Verlag, 2000

K E ATKINSON, *An Introduction to Numerical Analysis*, 2nd Edition, Wiley, 1989.

S D CONTE and C DE BOOR, *Elementary Numerical Analysis*, 3rd Edition, McGraw-Hill, 1980.

G M PHILLIPS and P J TAYLOR, *Theory and Applications of Numerical Analysis*, 2nd Edition, Academic Press, 1996.

W GAUTSCHI, *Numerical Analysis: An Introduction*, Birkhäuser, 1977

H.R. SCHWARZ, *Numerical Analysis: A Comprehensive Introduction*, Wiley, 1989