The higher-dimensional picture

And its role in isogeny-based cryptography

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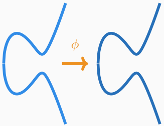
Isogeny-based cryptography

Candidate for post-quantum cryptography based on the hard problem of finding isogenies



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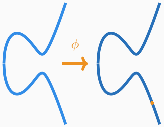
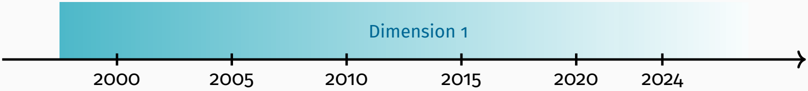


Dimensions in isogeny-based crypto



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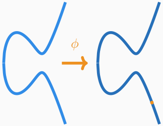

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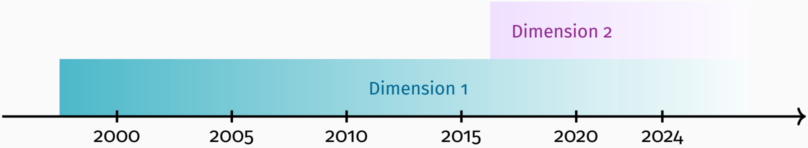

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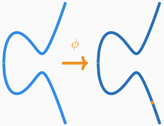

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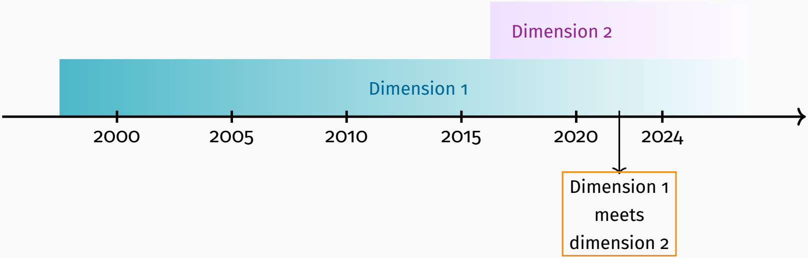

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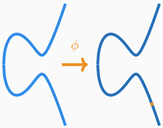

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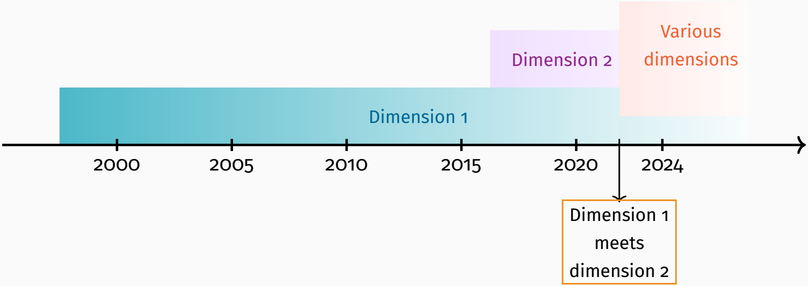

The 1-dimensional picture

Elliptic curves

An **Elliptic Curve** E over $\mathbb{F}_{p^k}$
is defined by an equation

$$E : y^2 = x^3 + ax + b,$$

where $4a^3 + 27b^2 \neq 0$.

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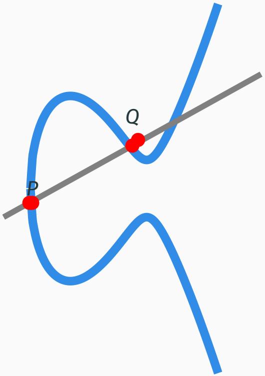

Points of E form an additive group.

- Can compute $P + Q$ for points $P, Q \in E(\mathbb{F}_{p^k})$.

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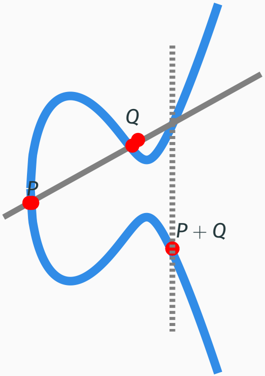

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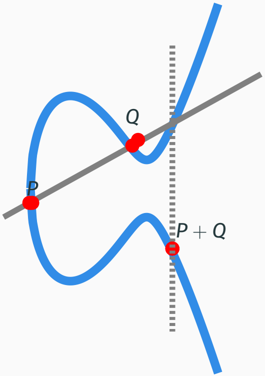

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- Can compute $m \cdot P$ for a point $P \in E(\mathbb{F}_{p^k})$ and $m \in \mathbb{Z}$.

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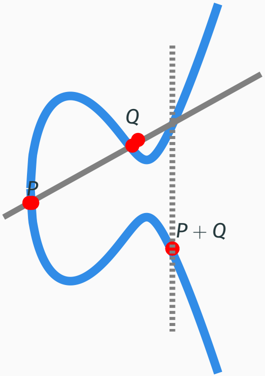

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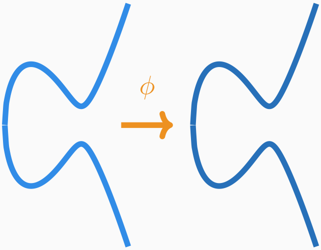
- Can compute $m \cdot P$ for a point $P \in E(\mathbb{F}_{p^k})$ and $m \in \mathbb{Z}$.

$\Rightarrow$ **One-way function:** $m \mapsto m \cdot P$ for some fixed $P \in E(\mathbb{F}_{p^k})$.

⚠ not a post-quantum one-way function

Isogenies



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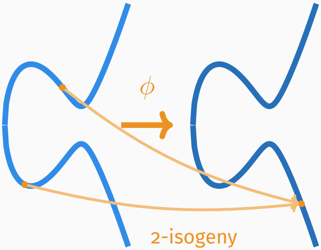
Isogenies



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An N**-isogeny** is an isogeny $\phi : E \to E'$ with kernel $K \simeq \mathbb{Z}/N\mathbb{Z}$.

- Complexity: $O(N)$ (Vélu) or $\tilde{O}(\sqrt{N})$ ($\sqrt{\text{é}}$lu)

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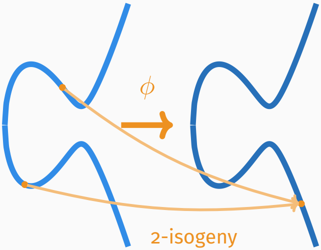

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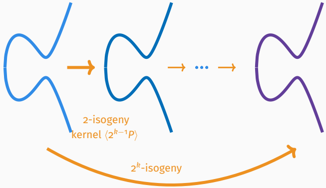
Smooth-degree isogenies

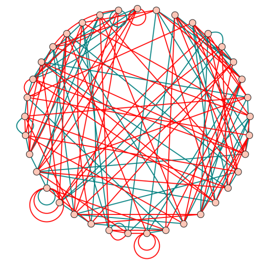
- Composition of small degree isogenies
- E.g. for $N = 2^k$ in time $O(k \log(k))$.



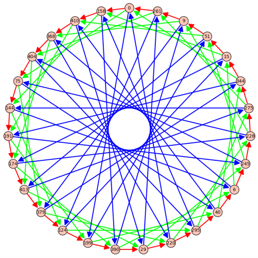
Isogeny graphs

- **Vertices**: elliptic curves (E).
- **Edges**: ℓ-isogenies with $\ell \in \{\ell_1, \ldots, \ell_n\}$ (E)—(E').

Two typical graphs



supersingular curves over $\mathbb{F}_{p^2}$
$\ell \in \{2, 3\}$, $p = 431$



supersingular curves over $\mathbb{F}_p$
$\ell \in \{3, 5, 7\}$, $p = 419$.

Isogenies as one-way functions

Setup Fix an elliptic curve E,
in an $\{\ell_1, \ldots, \ell_n\}$-isogeny graph with efficient navigation.

Isogeny one-way function

Input bit-string	$\rightsquigarrow$	path in the graph E — … — E'	$\rightsquigarrow$	Output E'

No polynomial quantum attacks are known.

Key exchange based on isogenies

Setup
Fix a starting curve E.

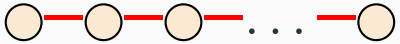
Key exchange based on isogenies

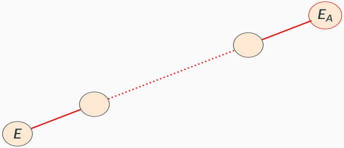
Setup

Fix a starting curve E.

Secret paths

Alice:





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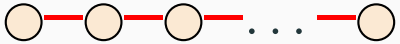
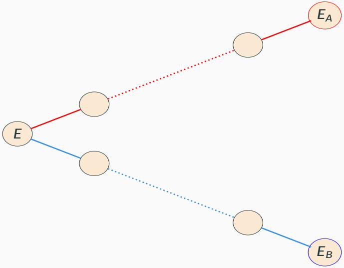


Bob:



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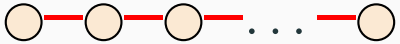
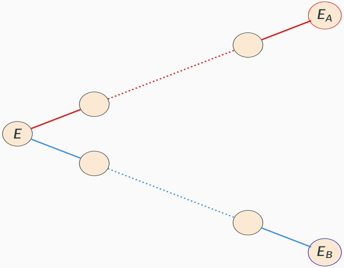


Bob:

Exchange

Alice $\xrightarrow{E_A}$ Bob

$\xleftarrow{E_B}$



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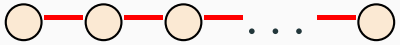

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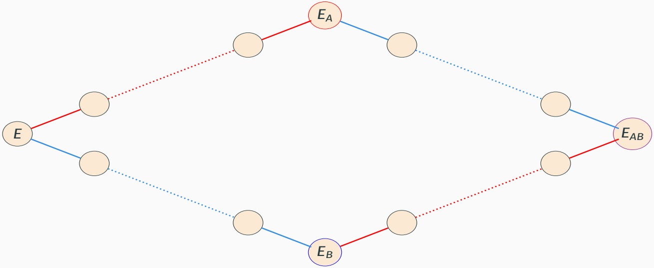
Shared key

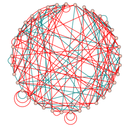
repeating* the path,

$\rightarrow$ (E_{AB})



(*) It is not obvious how to repeat a path with a different starting vertex, so that the paths commute.

Isogeny-based primitives in dimension 1



1997
Couveignes

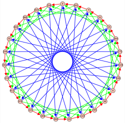
Hard homogeneous space
Group-action based cryptography
$\rightarrow$ DH key exchange with isogenies.

2006
Rostovtsev, Stolbunov

Public-key cryptosystem based on isogenies
Independent discovery of Couveigne's (unpublished) ideas.

2009
Charles, Goren, Lauter

CGL hash function Cryptographic hash functions from expander graphs.



2011
de Feo, Jao

SIDH
Towards quantum-resistant cryptosystems from supersingular elliptic curve isogenies

2018,
Castryck, Lange, Martindale, Panny, Renes

CSIDH:
an efficient post-quantum commutative group action

2020
de Feo, Kohel, Leroux, Petit, Wesolowski

SQISign:
compact post-quantum signatures from quaternions and isogenies

The 2-dimensional picture

What are 2-dimensional elliptic curves ?

An elliptic curve

- is a 1-dimensional variety

$$E : Y^2Z = X^3 + aXZ^2 + bZ^3 \subset \mathbb{P}^2.$$

- equipped with a group structure.

What are 2-dimensional ~~elliptic curves~~ principally polarized abelian varieties?

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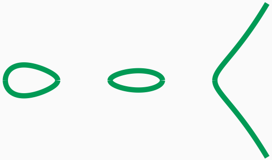
How to construct a p.p.a.v. of dimension 2?

$1 + 1 = 2$:
product of elliptic curves
$E_1 \times E_2$

$2 = 2$: Irreducible p.p.a.v of dimension 2

Genus-2 curve $C : y^2 = f(x),$ with $\deg(f) \in \{5, 6\}.$



$y^2 = x(x^2 - 1)(x^2 - 4)$

$2 = 2$: Irreducible p.p.a.v of dimension 2

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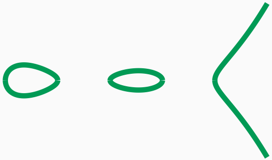

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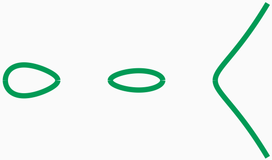

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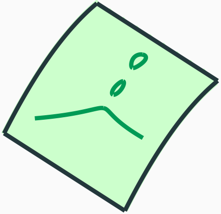
The **Jacobian of** C, $Jac(C)$, is a principally polarized abelian surface.

- Complicated as a variety (e.g. defined by 72 polynomials in $\mathbb{P}^{15}$).
- Easy description of $D \in Jac(C)$: $D = (P, Q)$ with P, Q points of C.



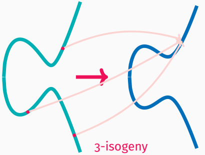
Isogenies in dimension 2

dimension 1

N-isogeny $\phi : E \to E'$ surjective morphism with $\ker(\phi) \simeq \mathbb{Z}/N\mathbb{Z}$.

dimension 2

(N, N)-isogeny surjective morphism $\phi : A \to A'$ has isotropic[1] $\ker(\phi) \simeq (\mathbb{Z}/N\mathbb{Z})^2$



[1] Weil pairing is trivial.

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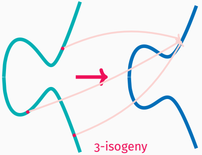

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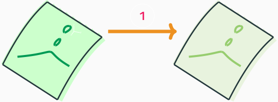

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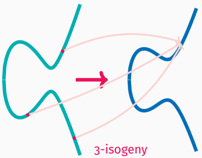

all isogenies are generic

dimension 2

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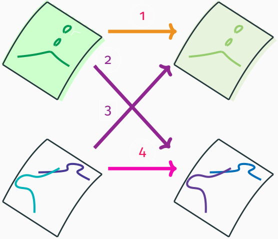


4 isogeny types:

1. generic
2. splitting
3. gluing
4. product

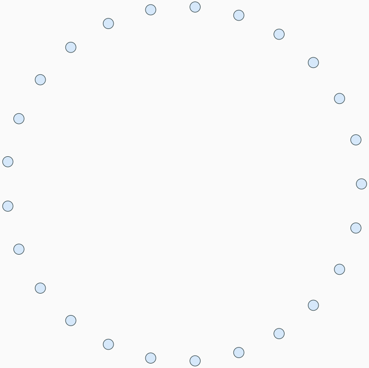
[1] Weil pairing is trivial.

Isogeny graphs in dimension 2

Vertices: p.p. abelian surfaces

○ $E \times E'$

(vey inaccurate) sketch of an isogeny graph



$\ell = 2$, $p = 53$

generically, the graph is 15-regular (for $\ell = 2$)

[a] More details on Slide 15

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 $E \times E'$

Edges: (ℓ, ℓ)-isogenies with

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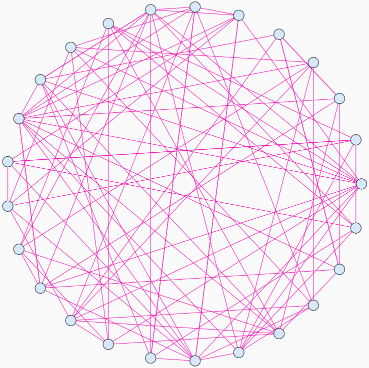

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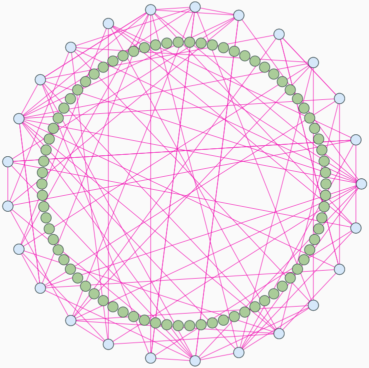

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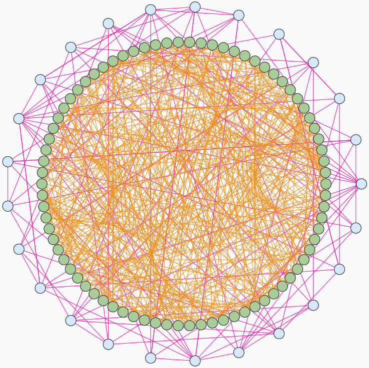

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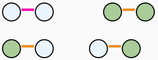
Edges: (ℓ,ℓ)-isogenies with $\ell \in \{\ell_1, \dots, \ell_n\}$



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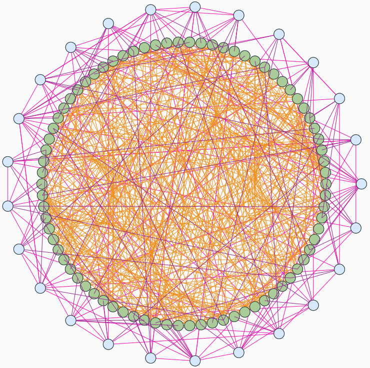

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Key features

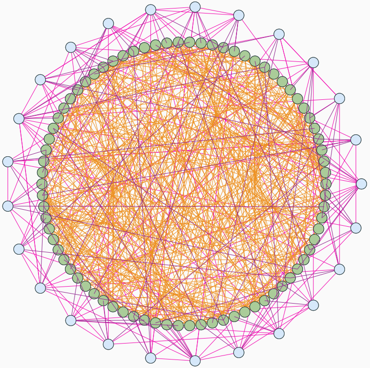
- # $\gg$ #;
- For small ℓ, we can navigate efficiently. [a]
- Finding a path

  is hard

[a] More details on Slide 15

(vey inaccurate) sketch of an isogeny graph



$\ell = 2$, $p = 53$

generically, the graph is 15-regular

(for $\ell = 2$)

Generalization of 1-dimensional crypto to dimension 2

Isogeny-based primitives in dimension 1

- 1997, Couveignes: CRS key exchange
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- 2009, Charles, Goren, Lauter: CGL hash function
- 2011, de Feo, Jao: SIDH key exchange
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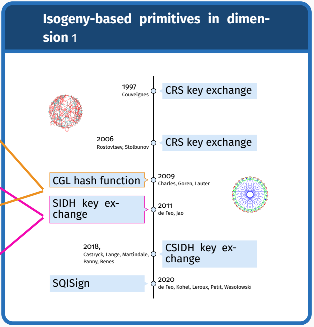

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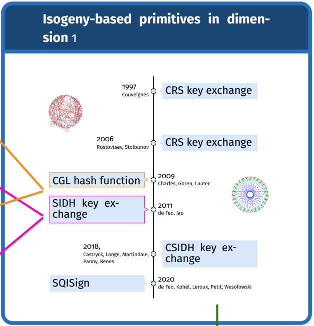
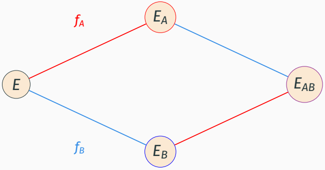


open problem

Dimension 2 meets dimension 1

Kani's Lemma (1997)

Isogeny diamond
(dimension 1)



d_A-isogeny f_A and d_B-isogeny f_B

$\Leftrightarrow$

Product isogeny
(dimension 2)

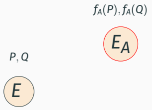
$(d_A + d_B, d_A + d_B)$-isogeny F

$d_A + d_B$ interpolation data of f_A, f_B $\Rightarrow$ kernel of F

The attacks on Supersingular Isogeny Diffie-Hellman (SIDH)

Kani's lemma serves as a key ingredient for attacking the isogeny one-way function **with torsion point information**.

Setting Given E, E_A and interpolation data $P, Q, f_A(P), f_A(Q)$ with $\langle P, Q \rangle = E[d_A + d_B]$, find f_A.



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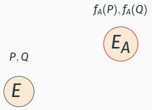

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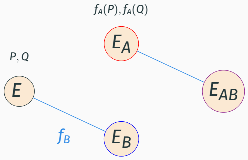

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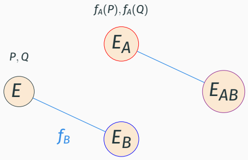

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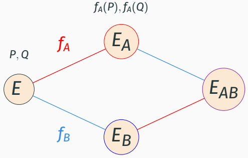
3. Recover f_A from F.



Computational aspects of the attacks

$d_A + d_B \in \{2^n, 3^m\} \Rightarrow$ need $(2,2)$- and $(3,3)$-isogenies

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$(3,3)$**-isogenies** $\rightarrow$ attack Alice's secret.

- First implementation (Decru-Kunzweiler '2023) optimizing formulas by Bruin-Flynn-Testa (2014)
- Formulas in theta coordinates (Costello-Santos-Smith '2024)

More dimensions!

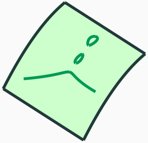
The 3-dimensional picture(s)

dimension 1
(abelian curves)

elliptic curve

dimension 2
(abelian surfaces)

product of elliptic curves



Jacobian of a genus-2 curve

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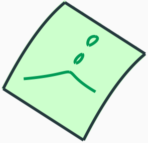

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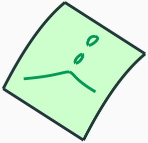

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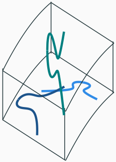

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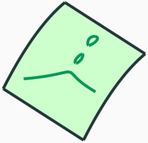

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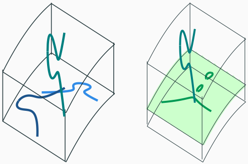

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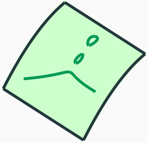

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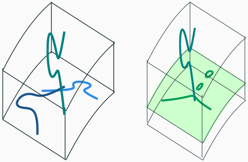

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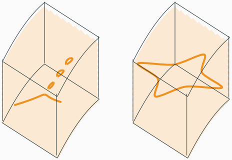

Jacobians of genus-3 curves

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1. **for cryptanalysis**: The unconditional poly-time attack on SIDH (Robert) requires working in dimension 8.

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$\Rightarrow$ new tool: HD representations!

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Any N-isogeny $f : E \to E'$ (of elliptic curves) has an efficient representation in dimension $d \in \{2, 4, 8\}$.
$\Rightarrow$ Evaluation in $O(\log^c(N))$ for some constant c.

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2. **for constructive applications:**

- SQISignHD
- SQISign2D $\times 3$
- FESTA, QFESTA
- IS-CUBE
- POKE
- SCALLOP-HD
- HD VRF
- CLAPOTIS

since 2022!

Computations in arbitrary dimensions

A: principally polarized abelian variety of dimension g.

- ✗ Dimension $g > 3$: A generically not the Jacobian of a curve.
- ✓ The Kummer variety $K = A/\langle \pm 1 \rangle$ has a nice representation:

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- $\ell = 2$: Algorithm by Robert (2023) in any dimension.
  - ✓ Implementations by Dartois, Maino, Pope, Robert ($g = 2$) and Dartois ($g = 4$)
  - ✗ dimensions $g = 3$ and $g > 4$ missing.
- $\ell \neq 2$ prime: Algorithms by Cosset, Lubicz, Robert in $\tilde{O}(\ell^g)$.
  - ✗ not yet optimized for crypto applications.

Conclusion

Exciting time for higher dimensions in isogeny-based cryptography.

What's next?

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- More applications of HD-representations.
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Thanks for your attention!