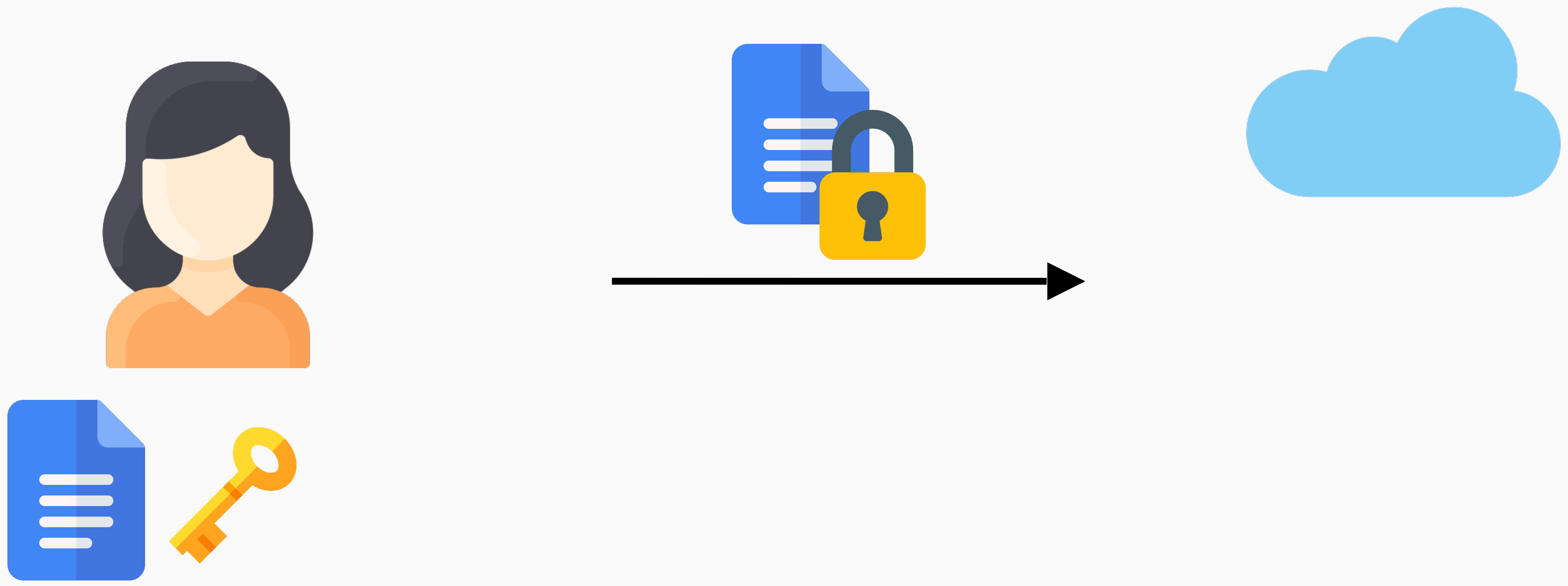
CCA Secure Updatable Encryption from Non-Mappable Group Actions

Jonas Meers, Doreen Riepel

June 13, 2024

Motivation

Cloud Storage



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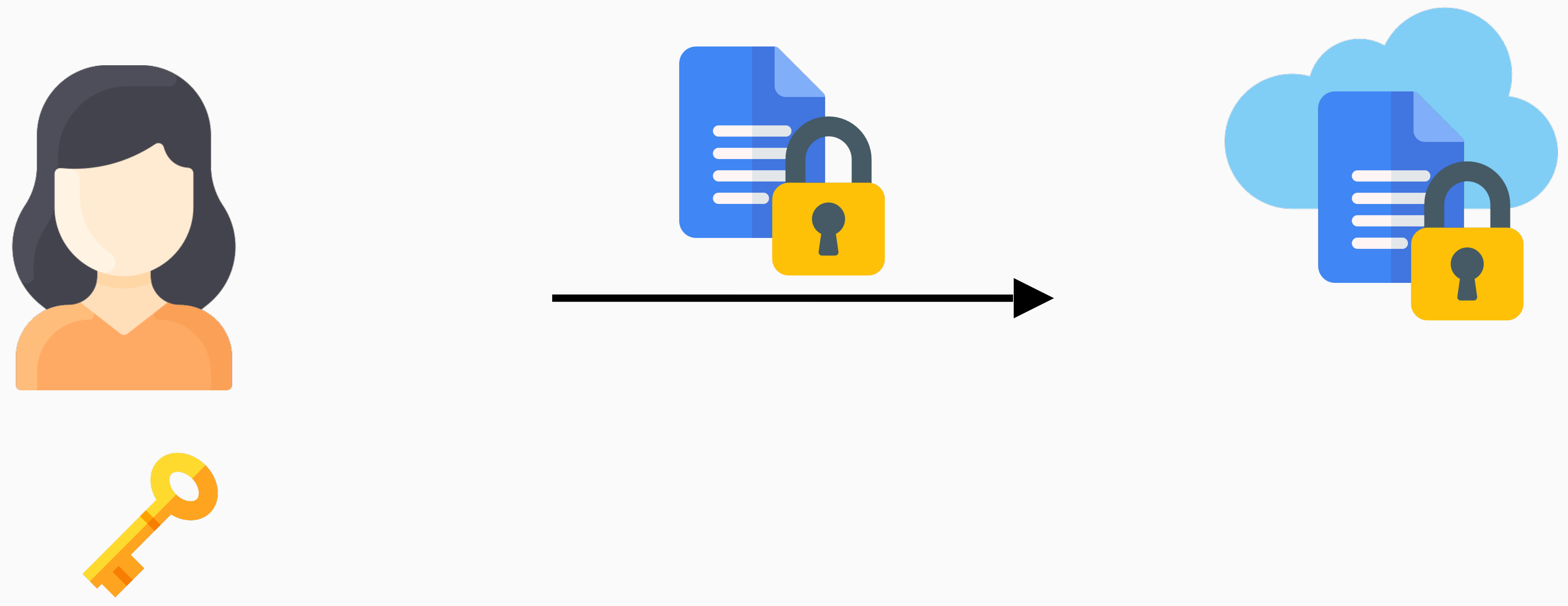

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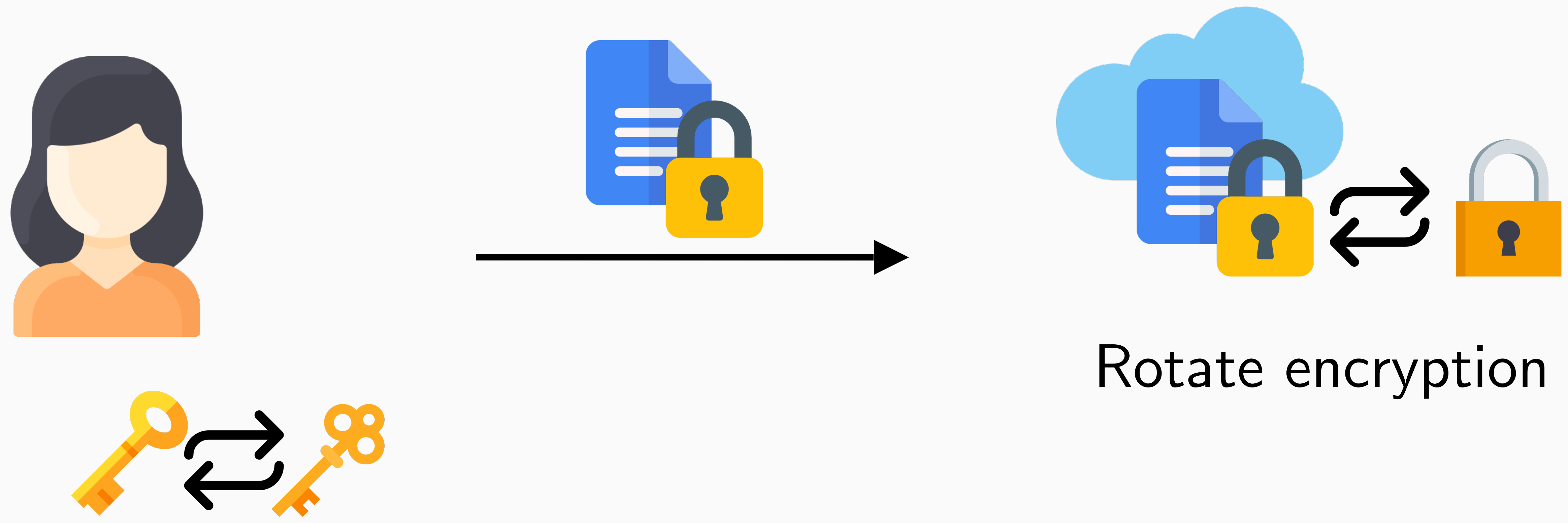

Rotate encryption keys

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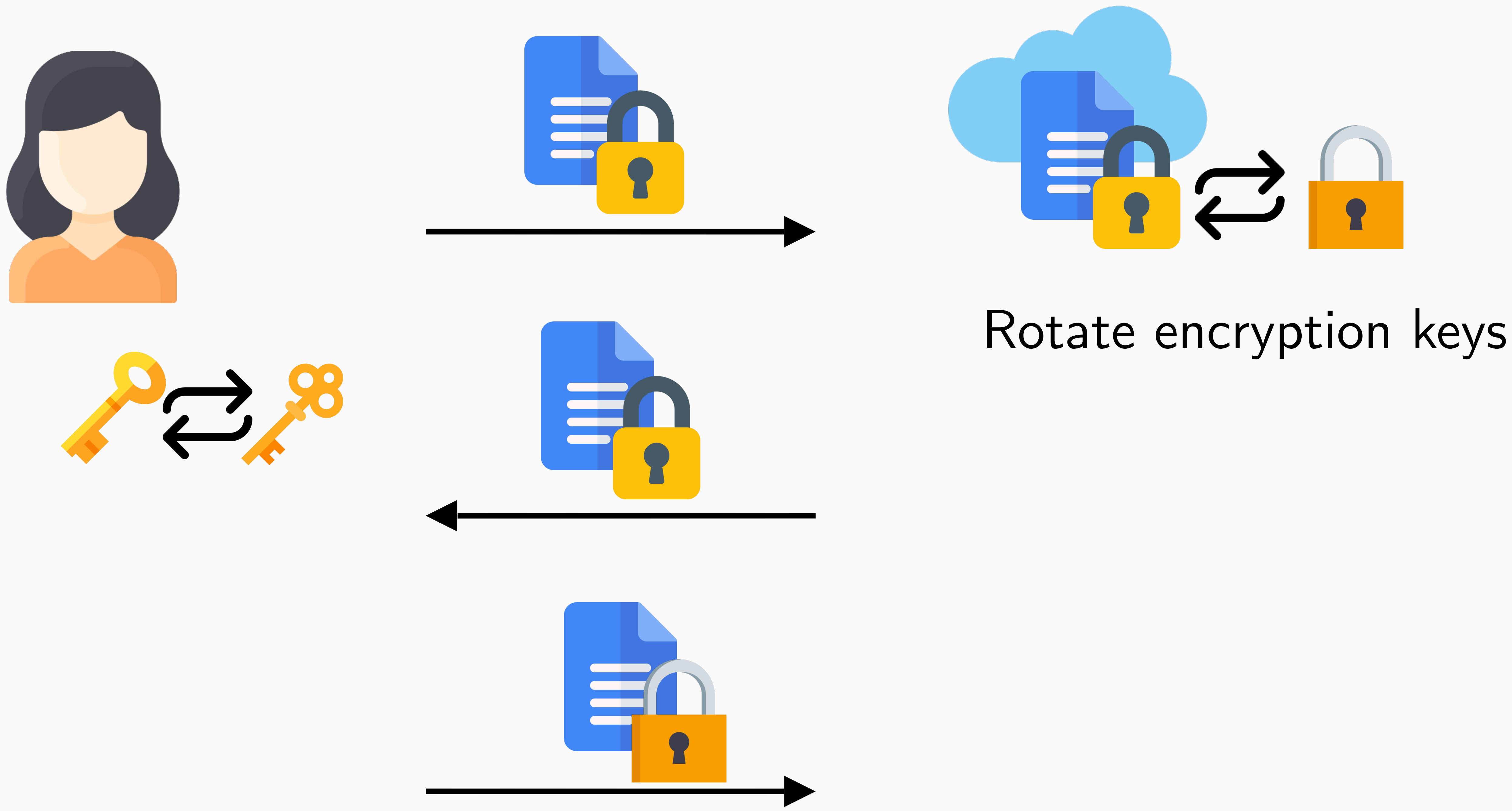

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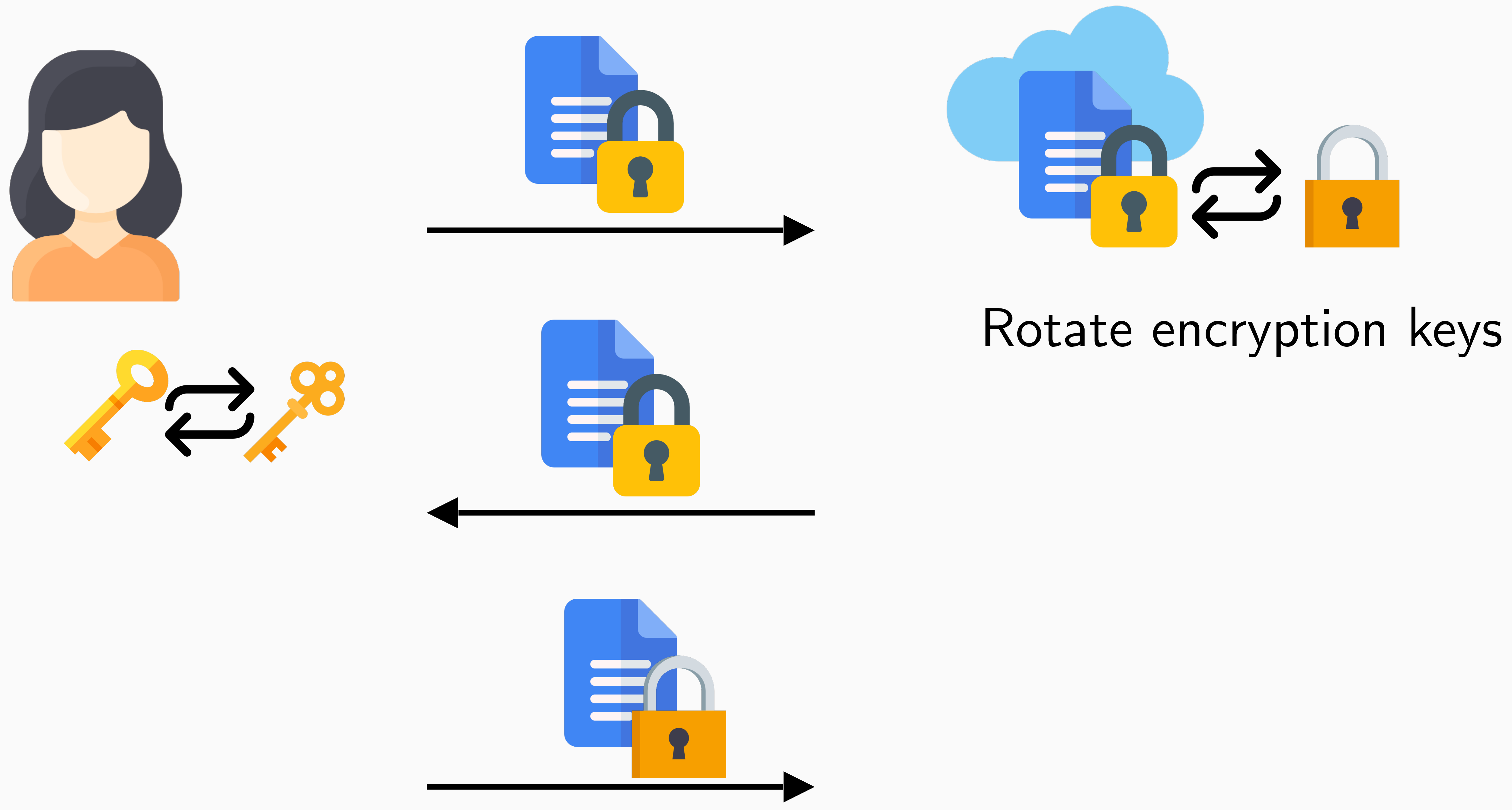

Inefficient!

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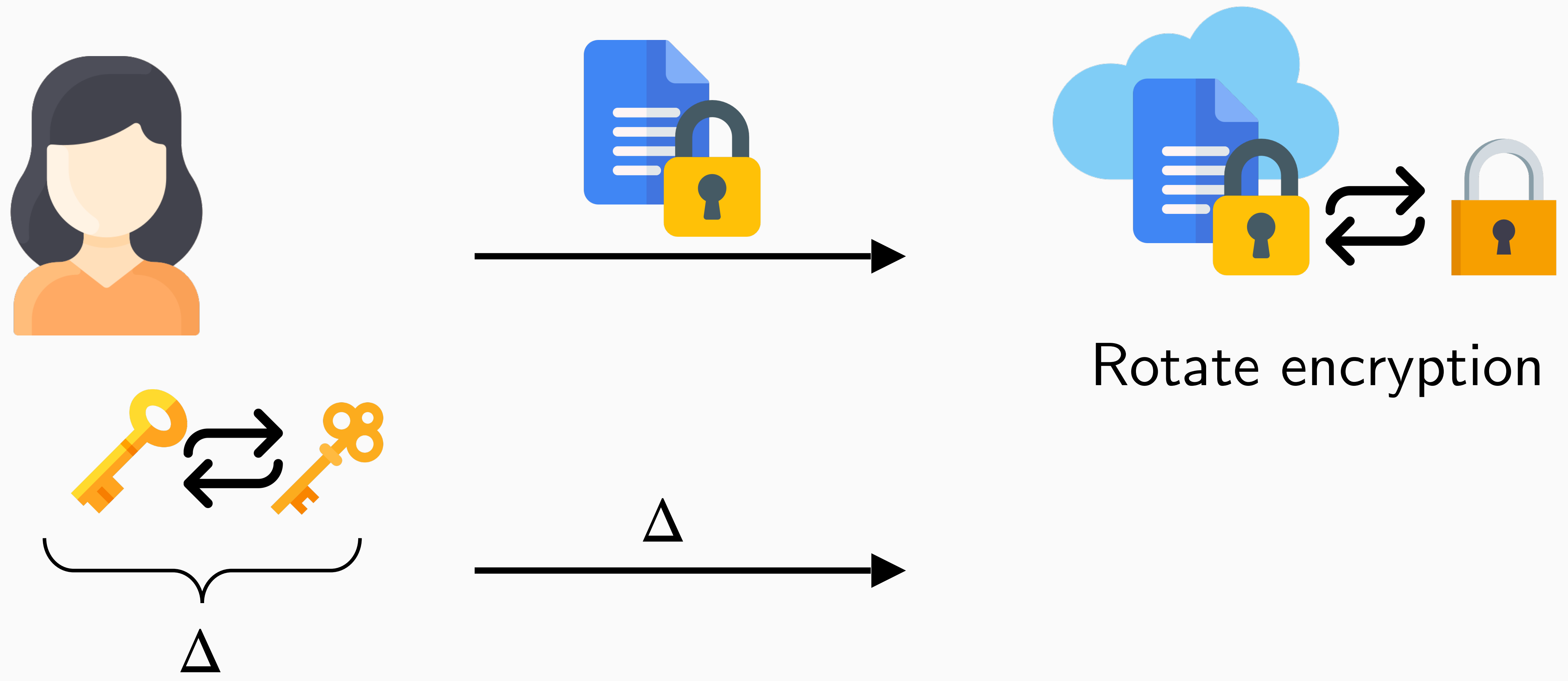

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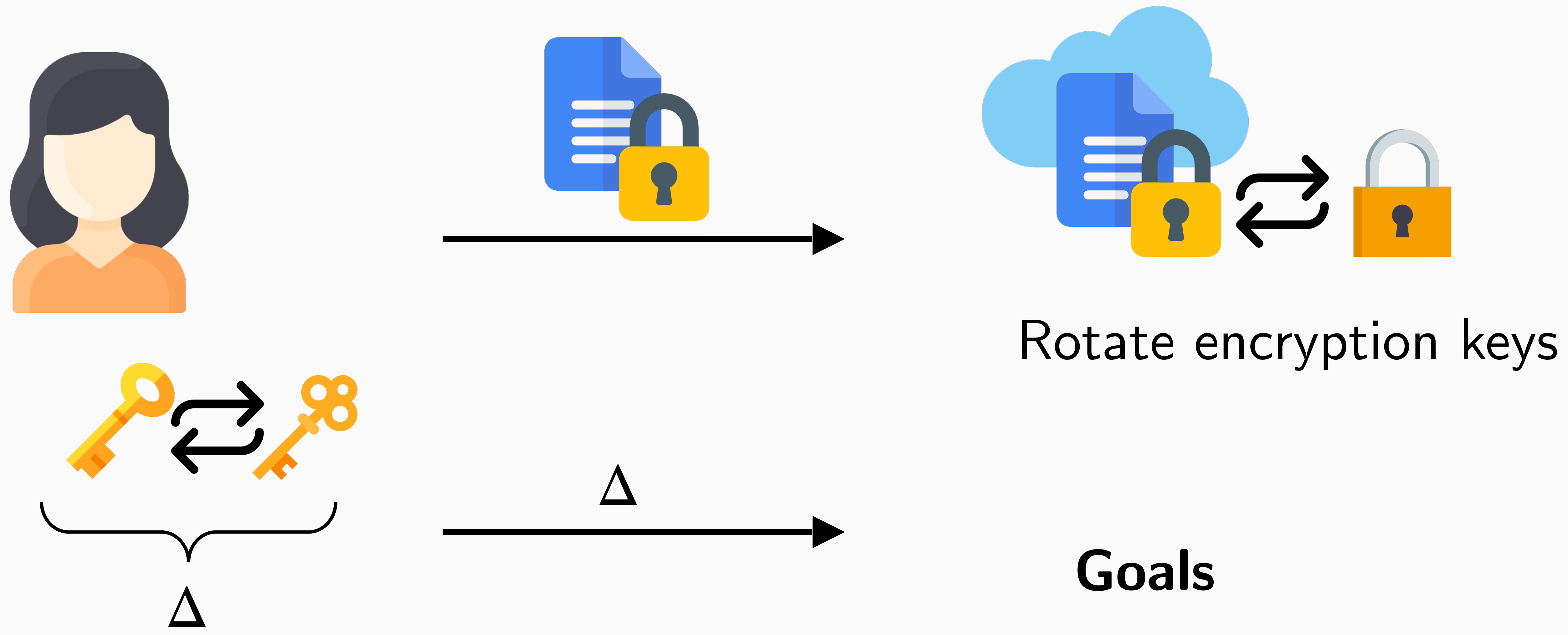

Goals

- **Confidentiality:** Cannot distinguish encryptions of two chosen messages
- **Integrity:** Cannot modify ciphertexts
- **Unlinkability:** Cannot tell which ciphertext an update was derived from
- **Forward secrecy:** Old ciphertext is secure even if current key leaks
- **Post-compromise security:** Old key does not help to decrypt updated ciphertext

Syntax

Updatable Encryption $\mathsf{UE} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{TokenGen}, \mathsf{Upd})$

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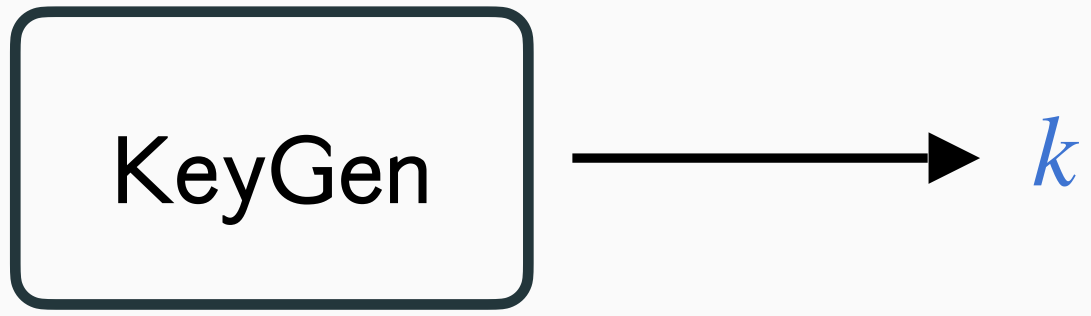

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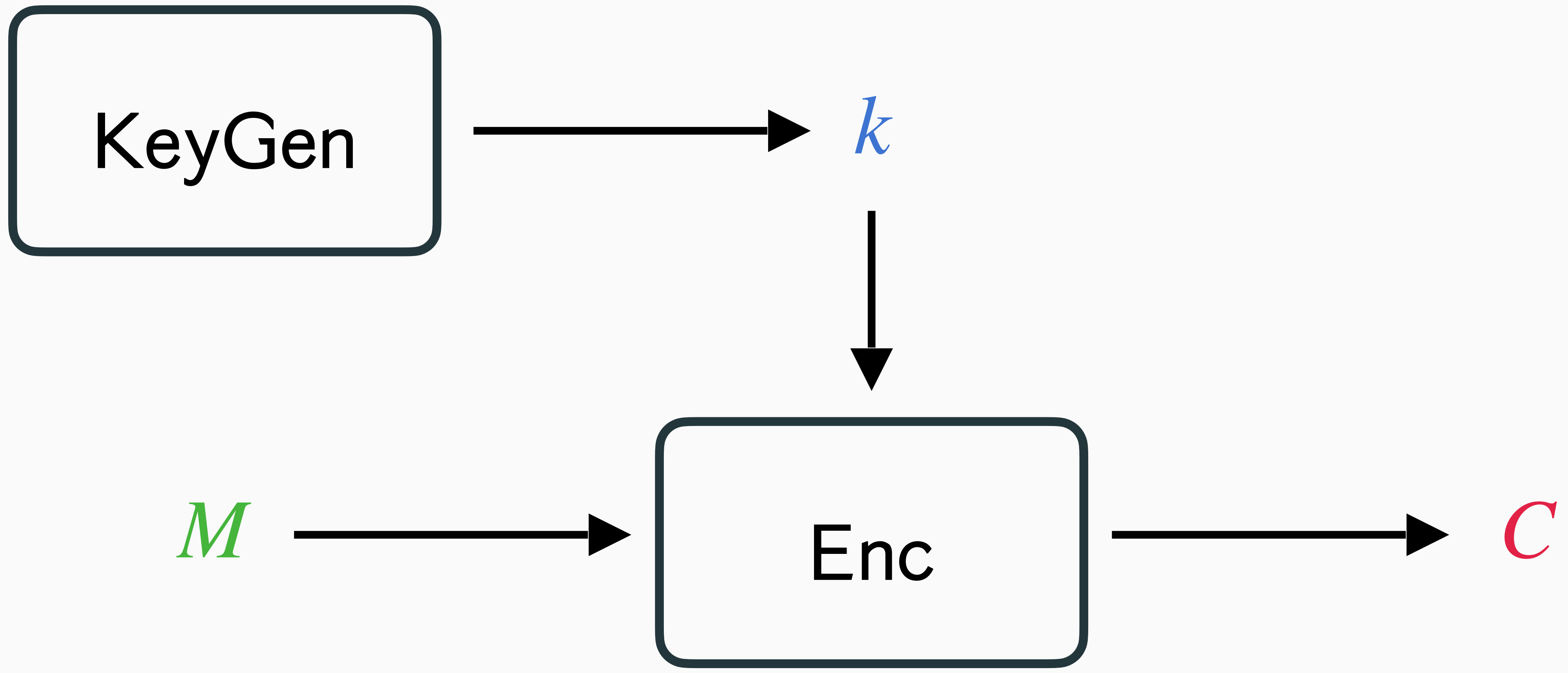

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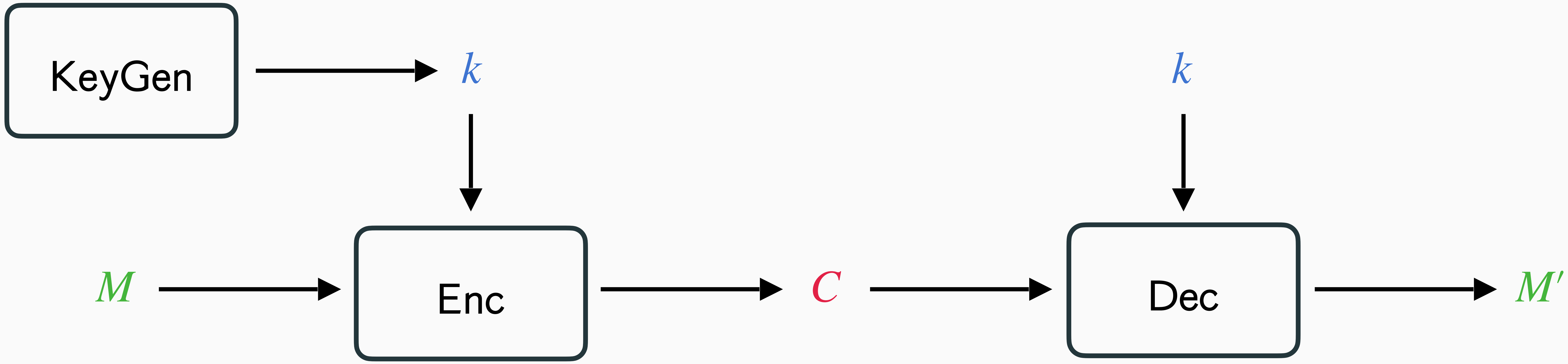

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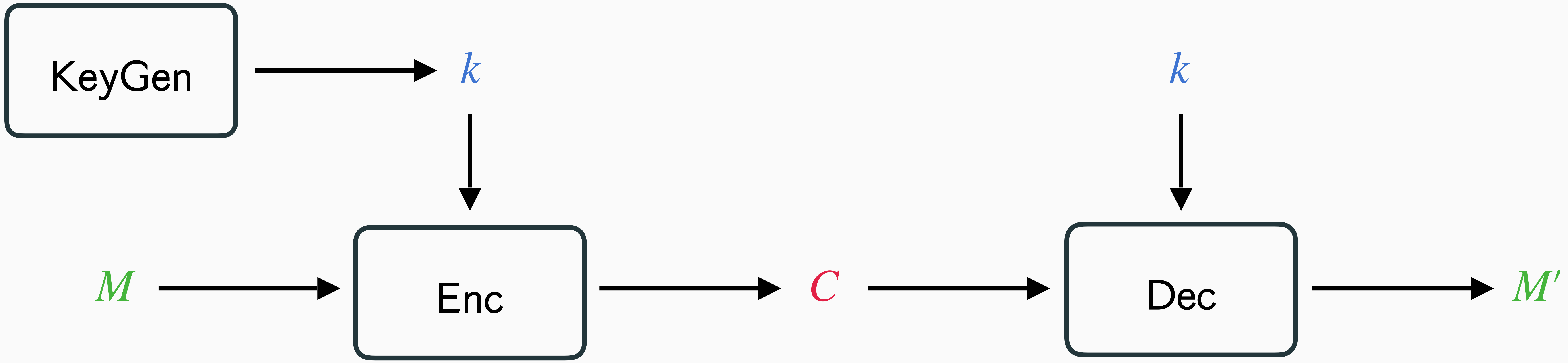

Correctness: $M = M'$

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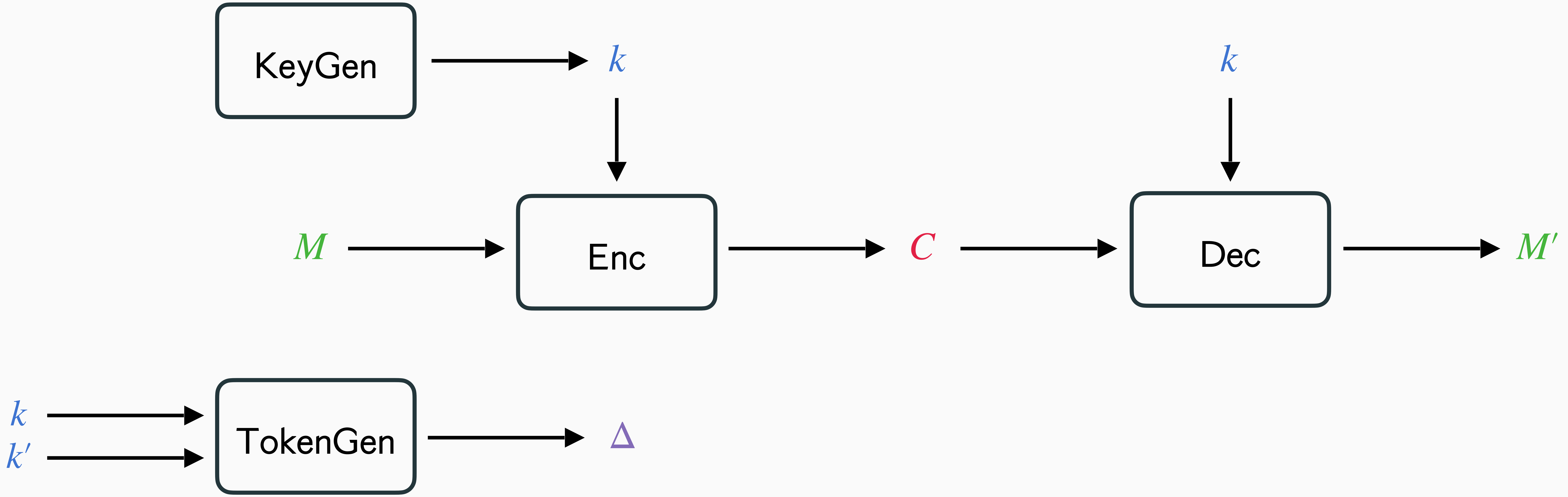

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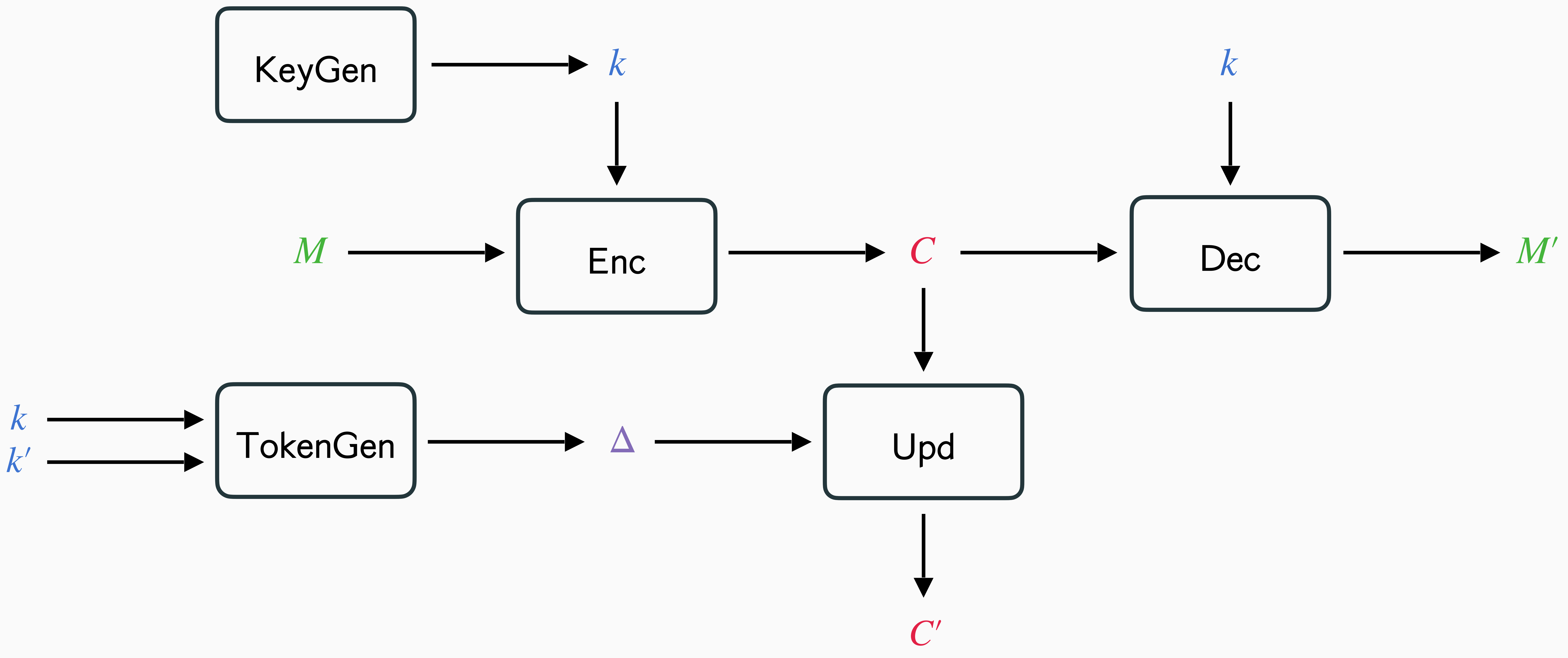

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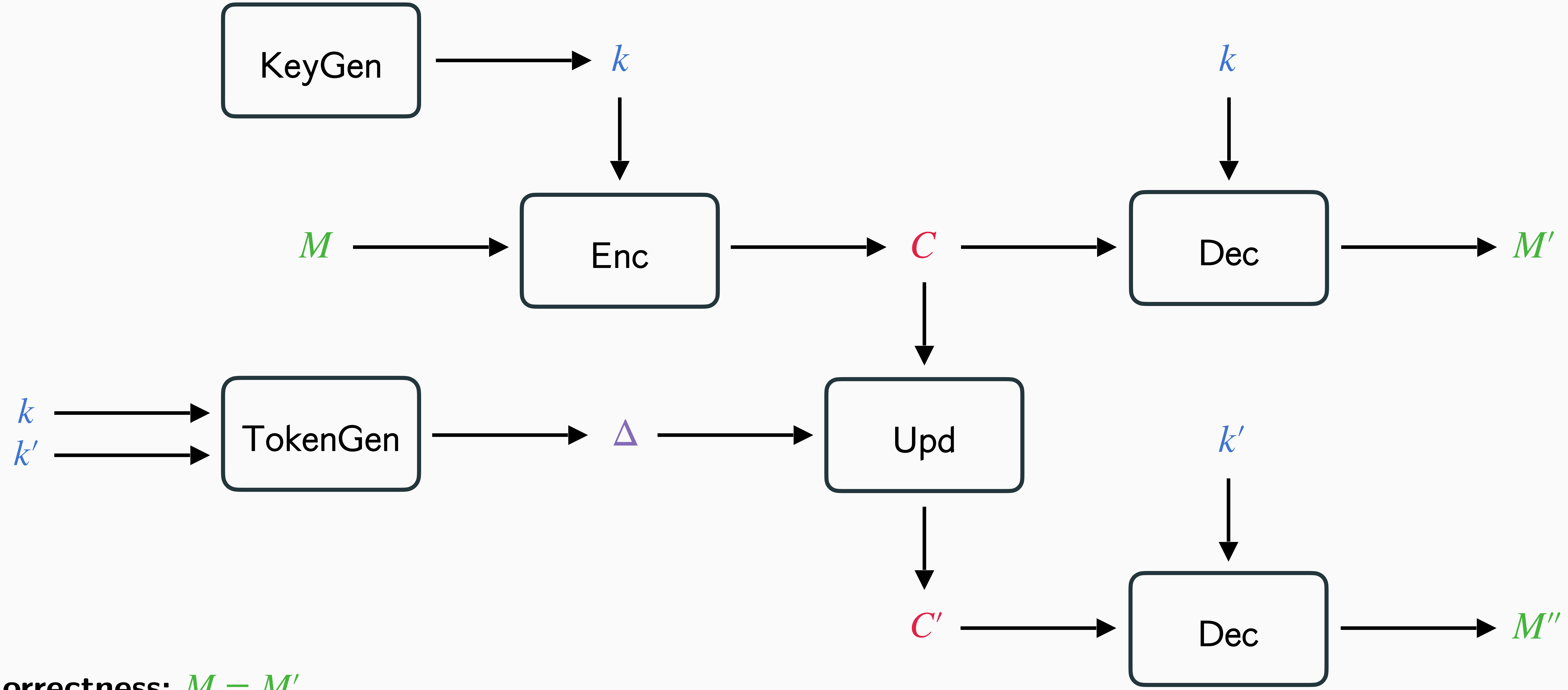

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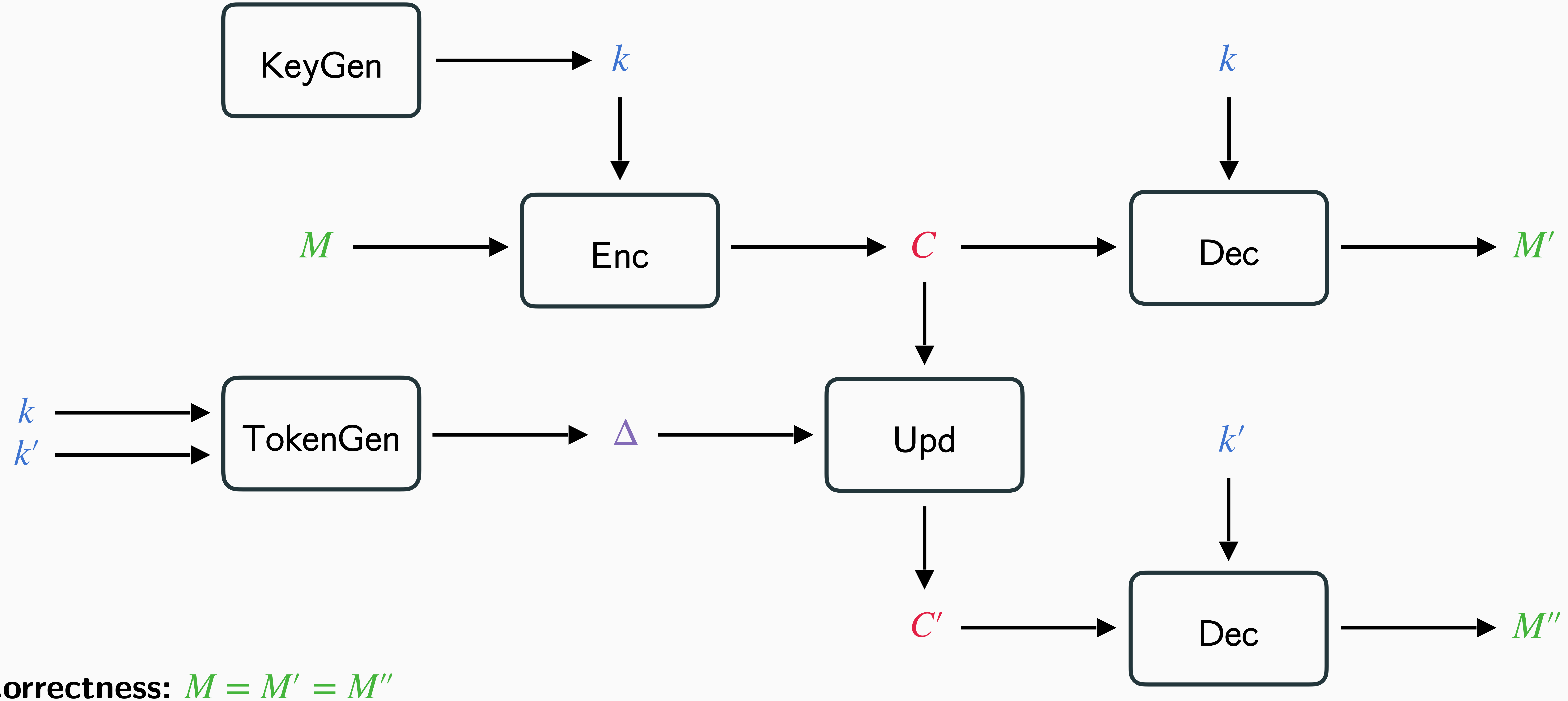

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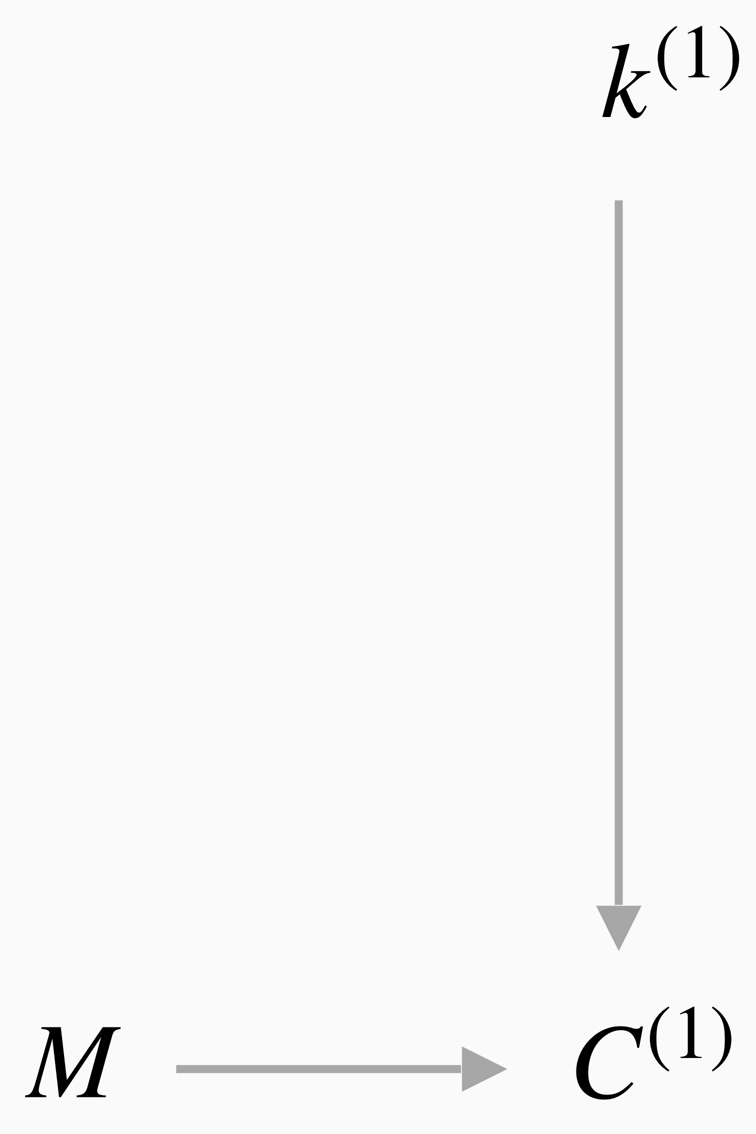
Modeling Security



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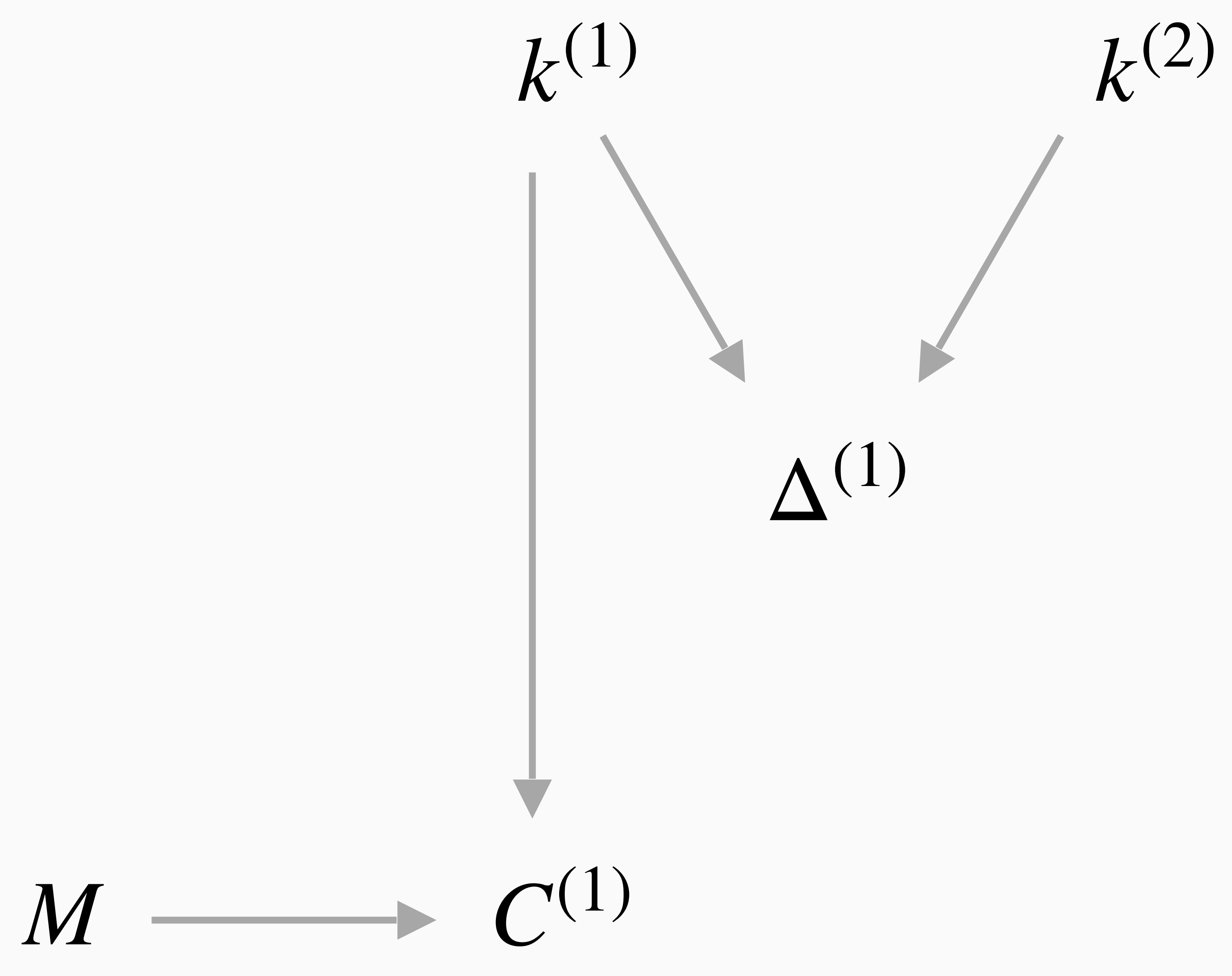

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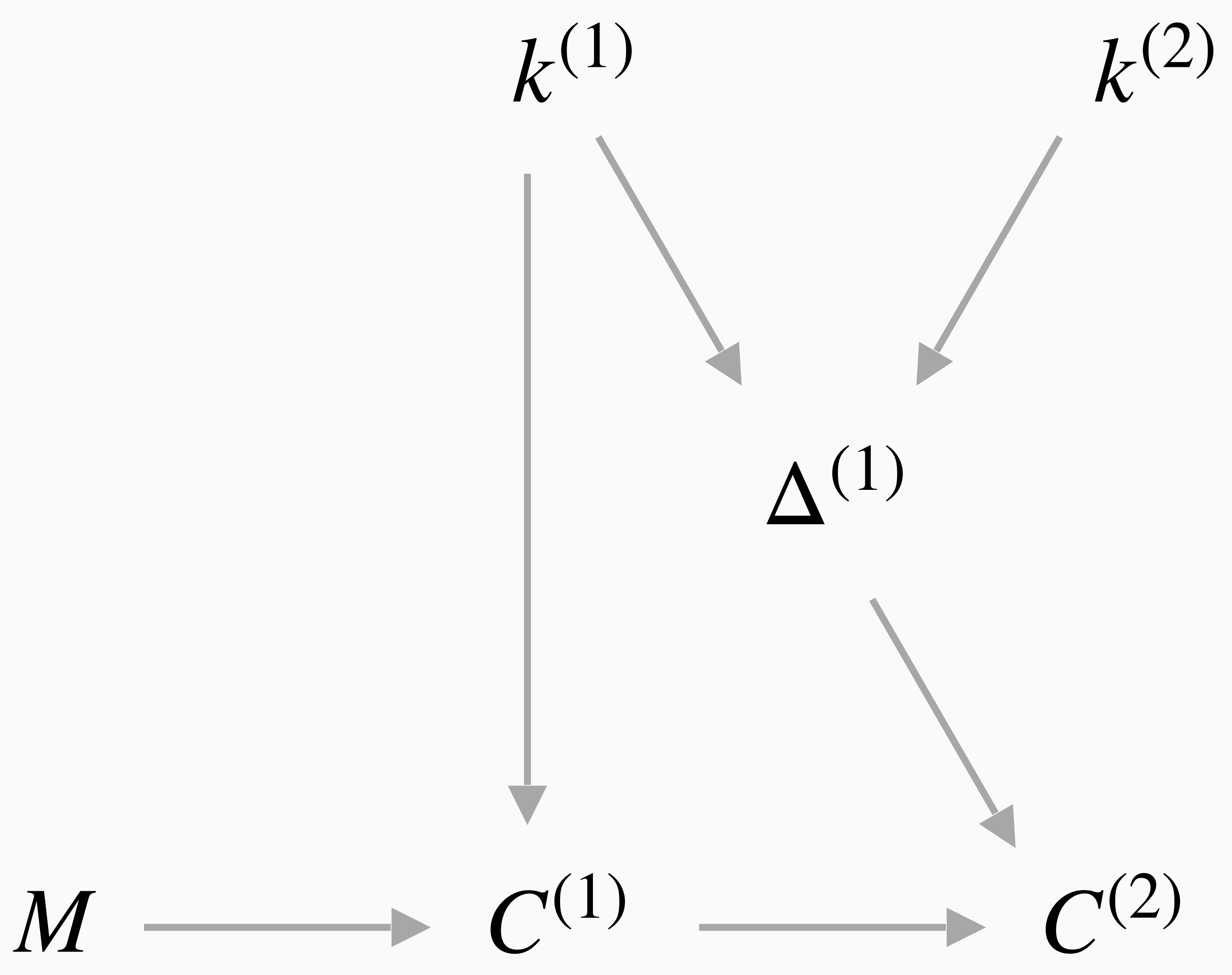

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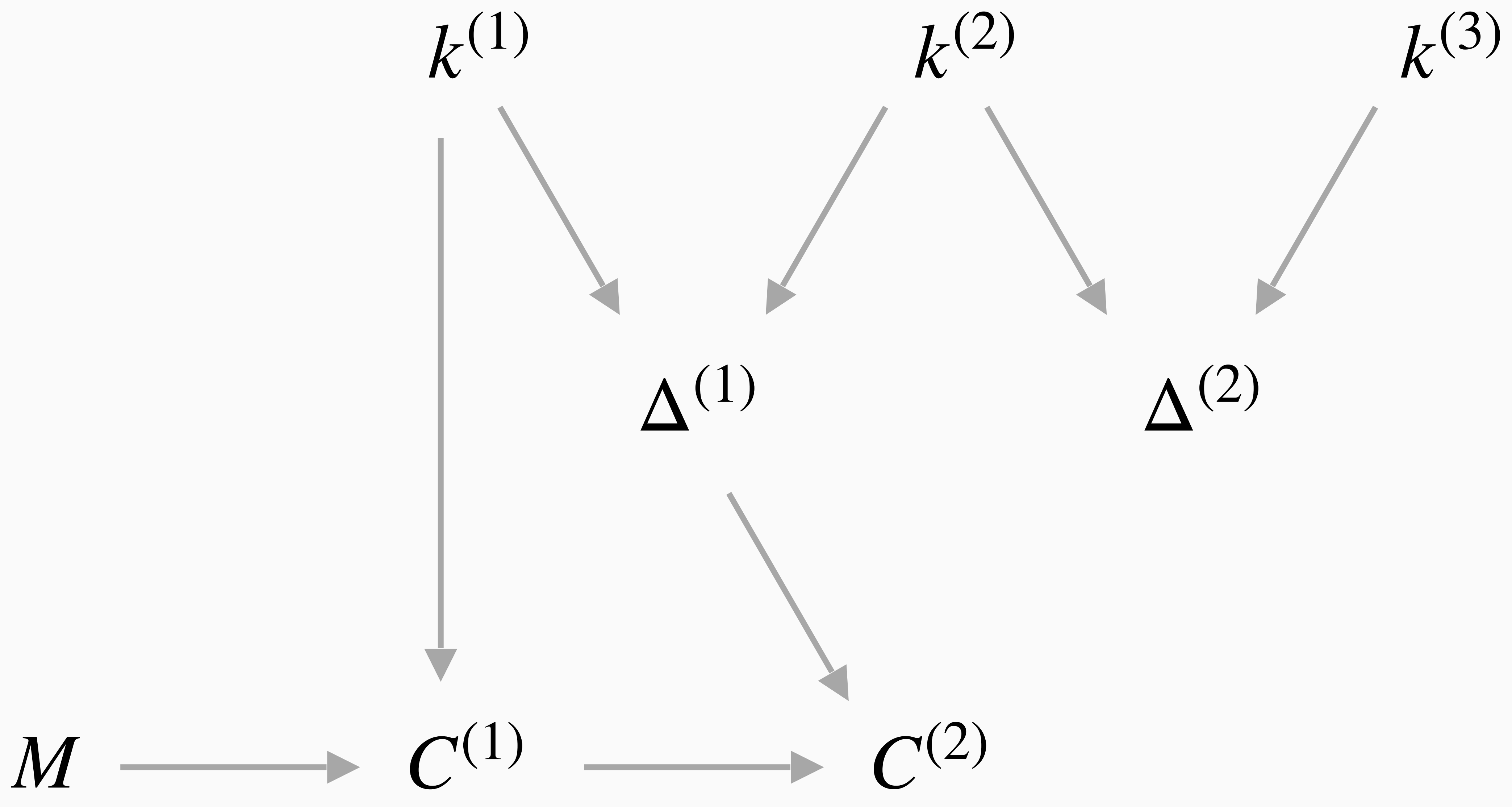

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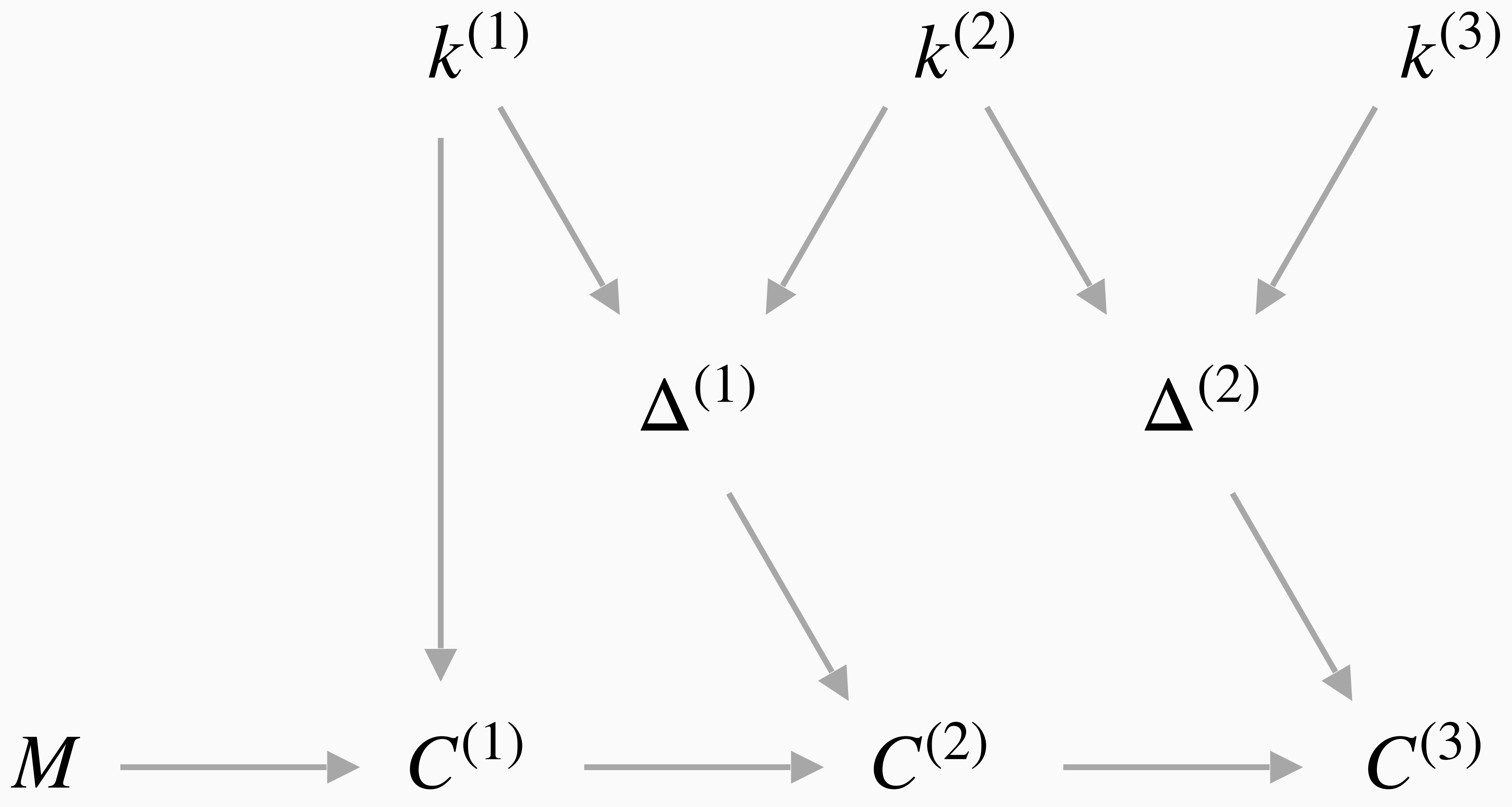

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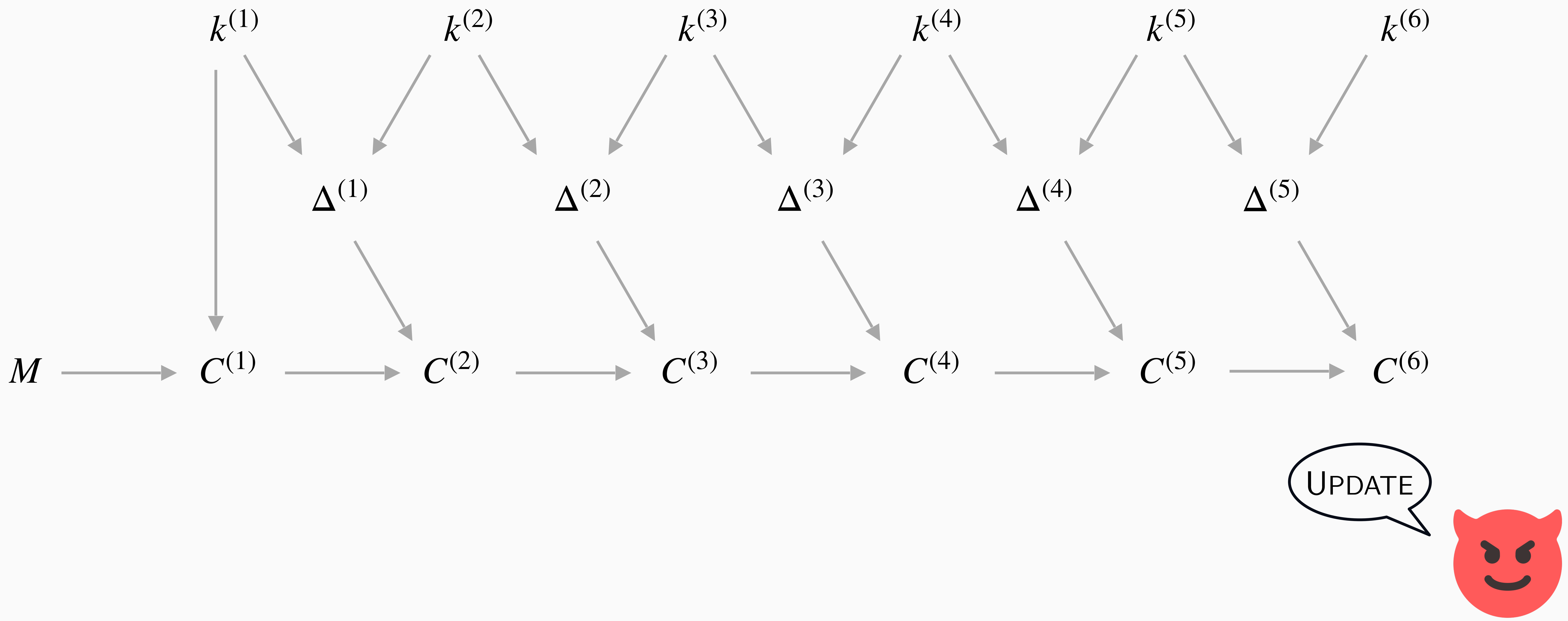

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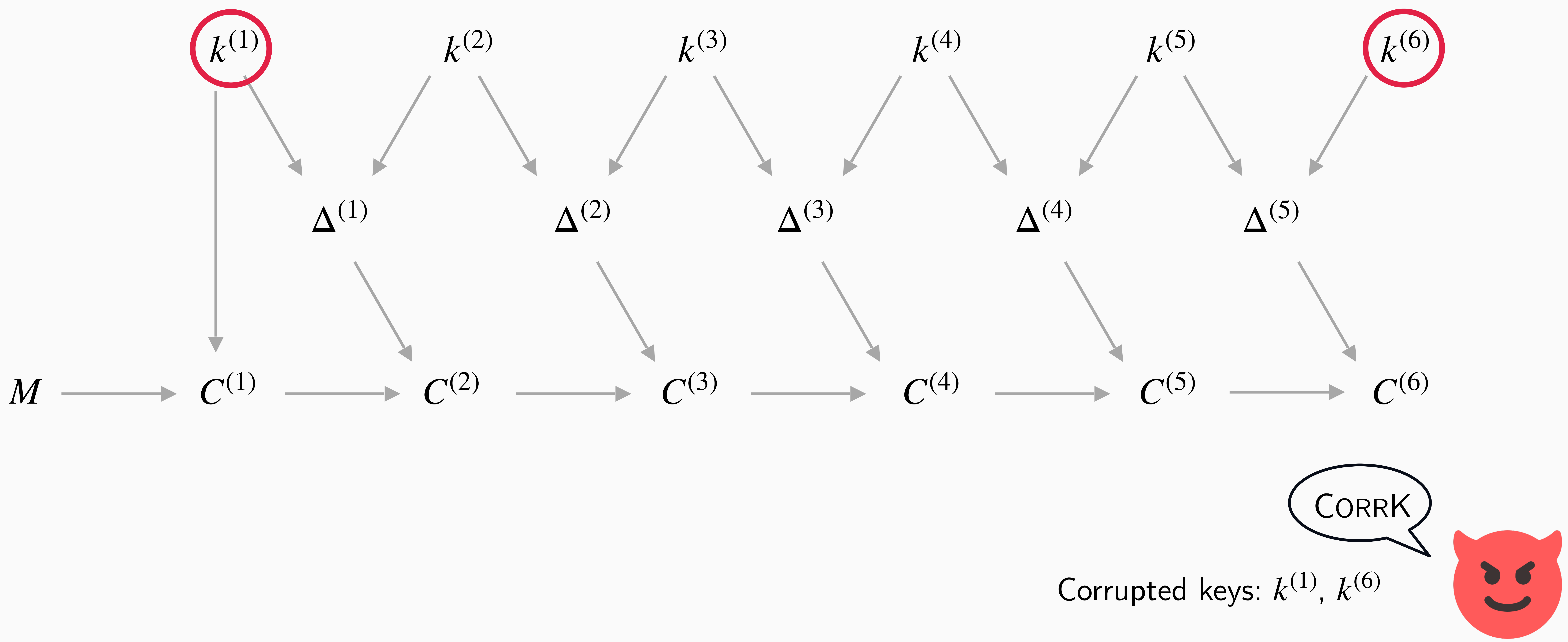

Corrupted keys: $k^{(1)}$, $k^{(6)}$

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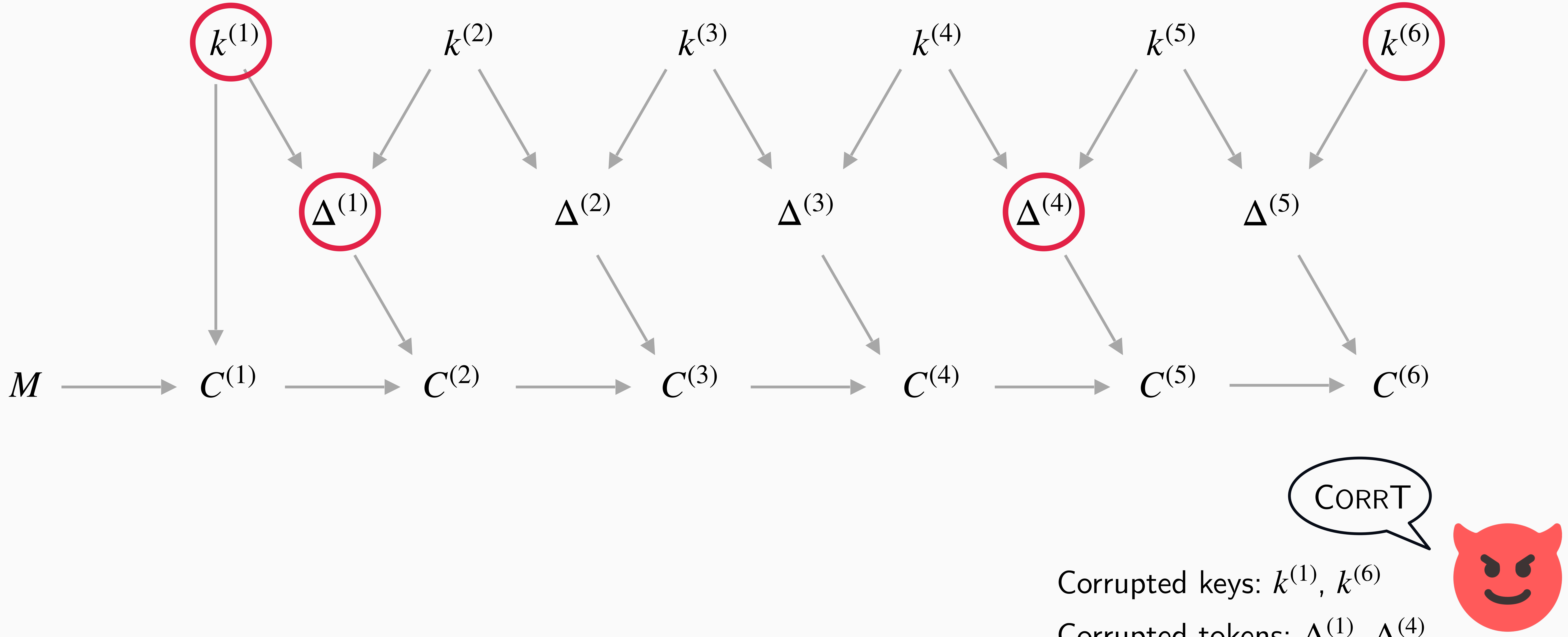

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Corrupted tokens: $\Delta^{(1)}$, $\Delta^{(4)}$

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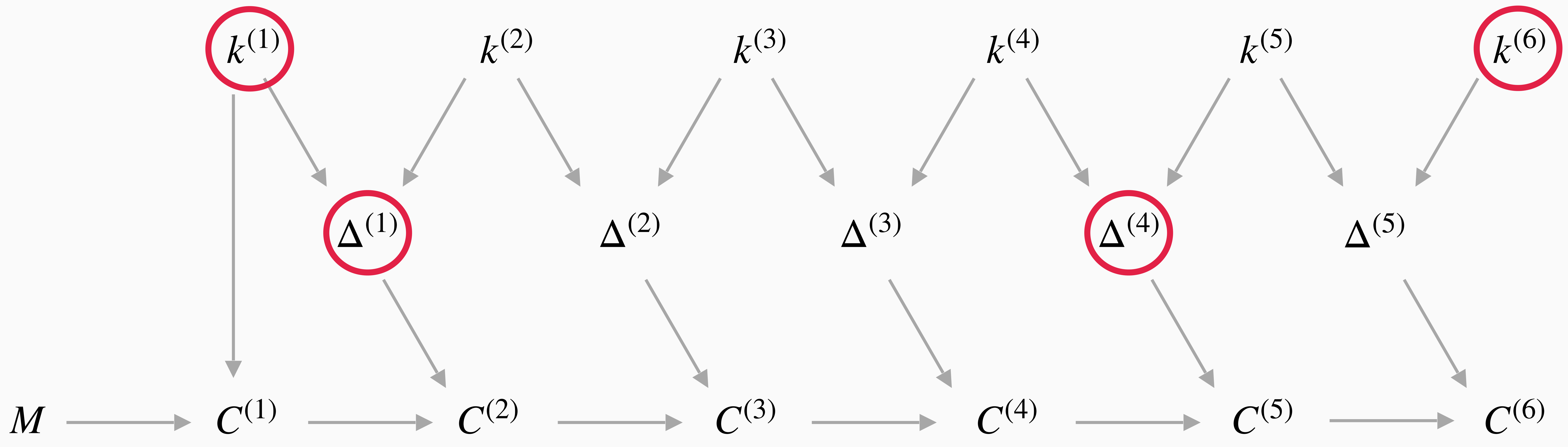

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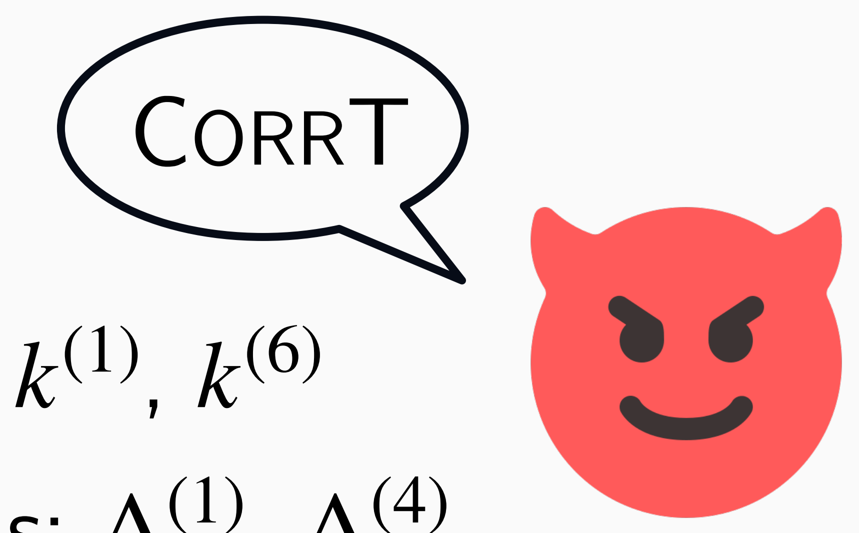
- Randomized or deterministic ciphertext updates
- Bi-directional and uni-/no-directional key updates
- CPA and (R)CCA security



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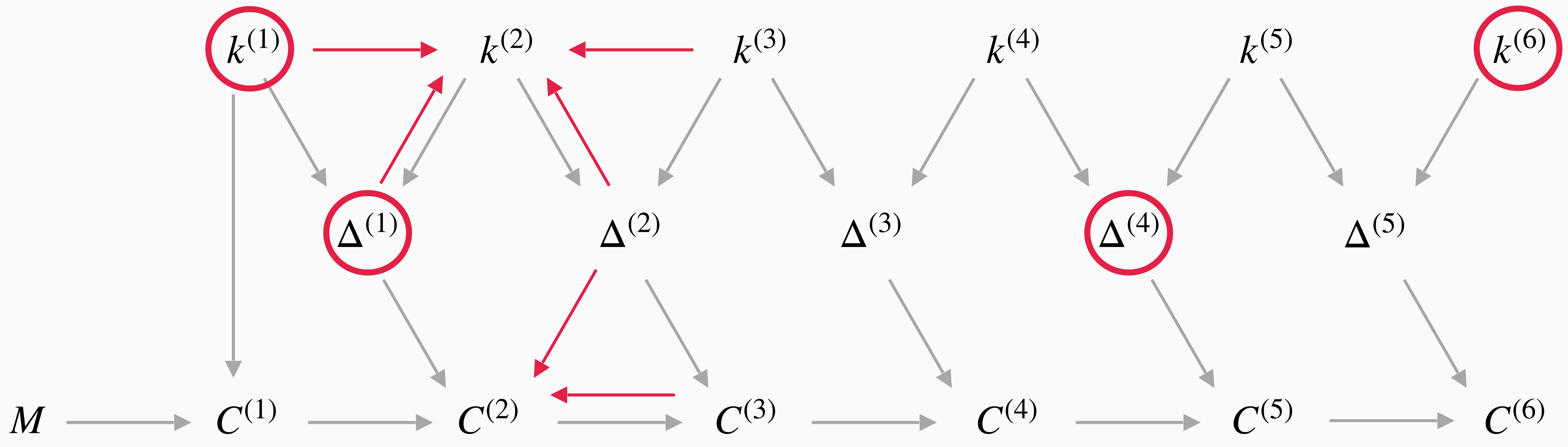

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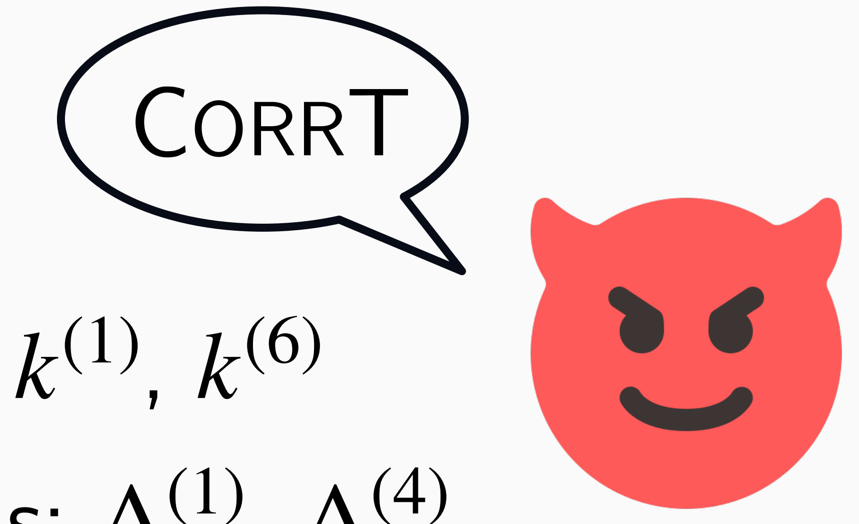

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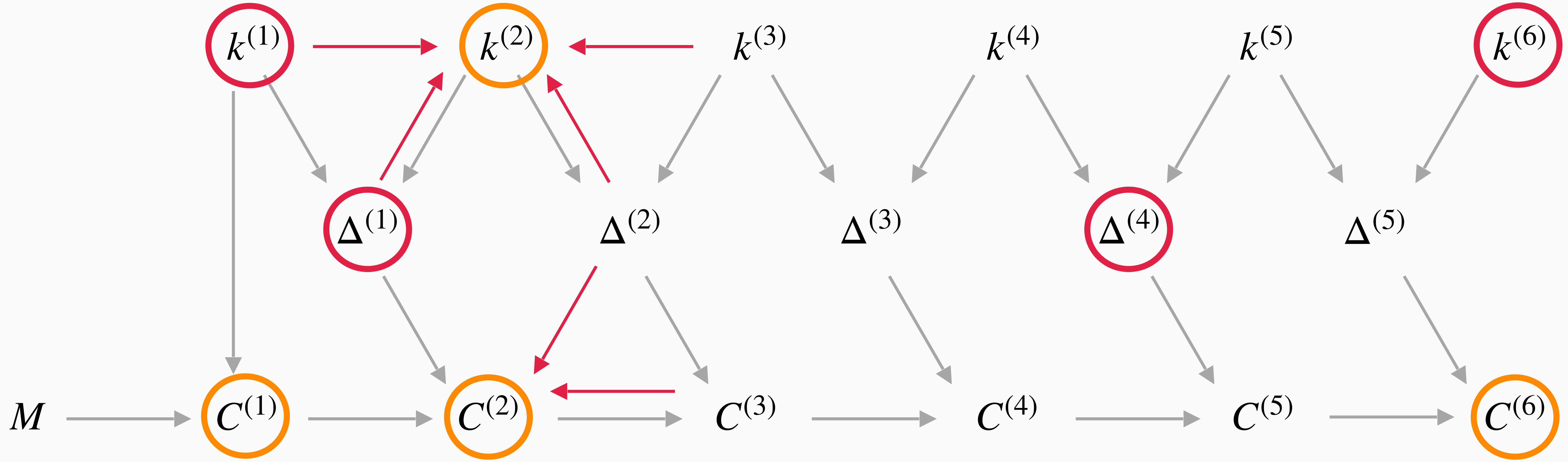

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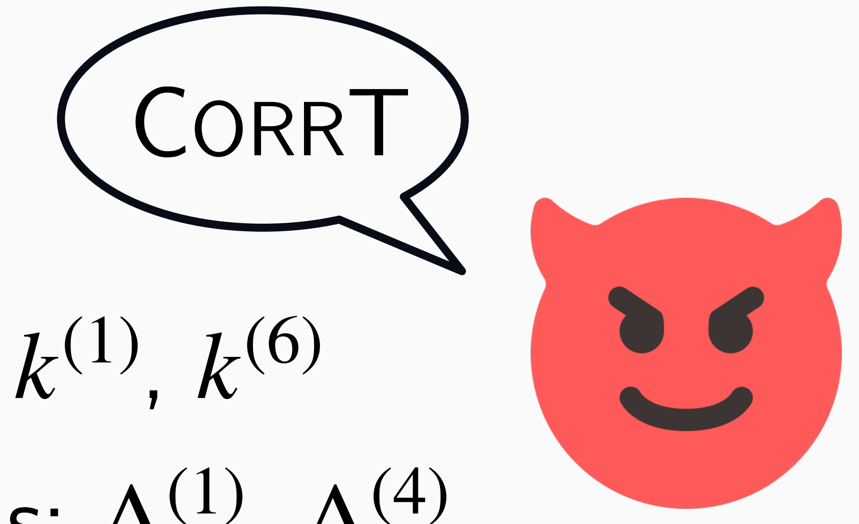

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Inferred knowledge: $k^{(2)}$, $M^{(1)}$, $M^{(2)}$, $M^{(6)}$

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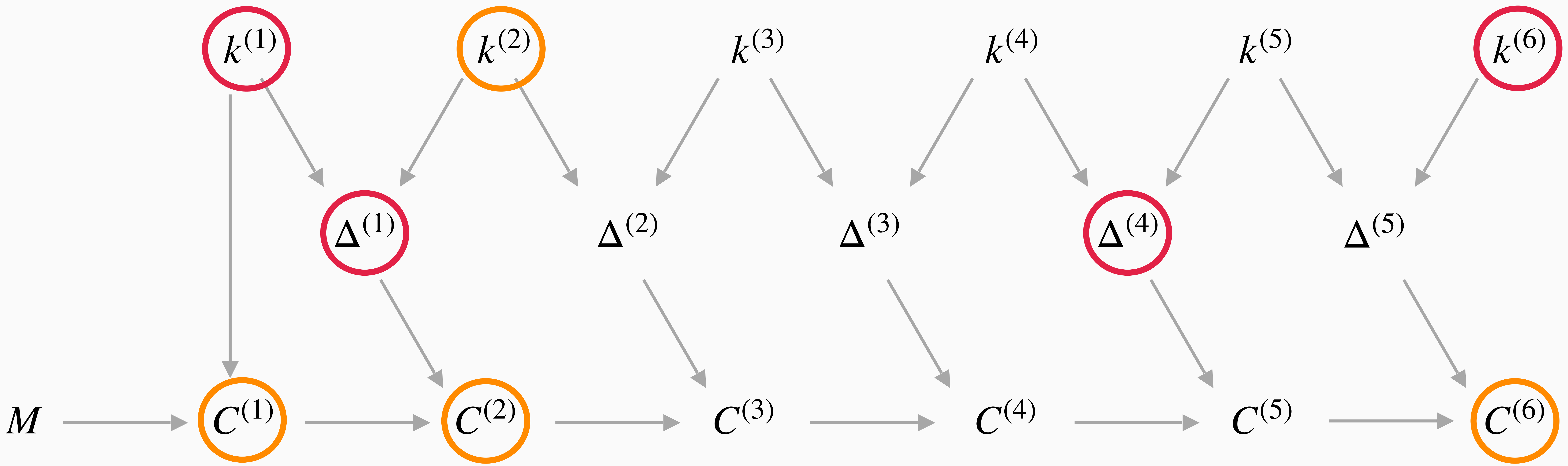

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- Distinguish updated ciphertext from encryption of a new message

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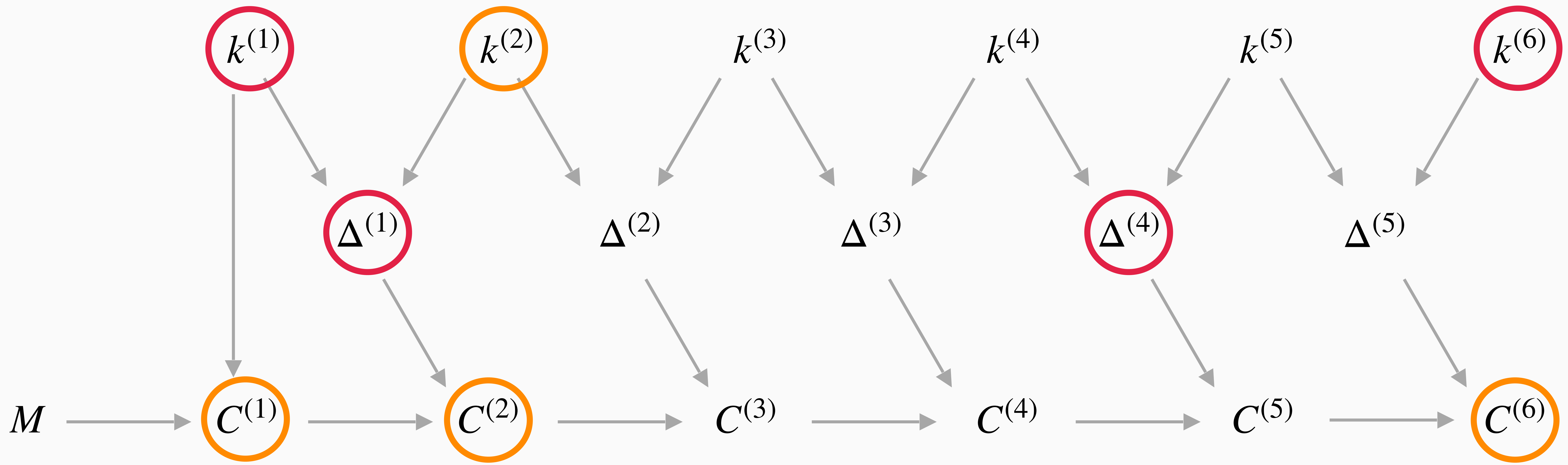

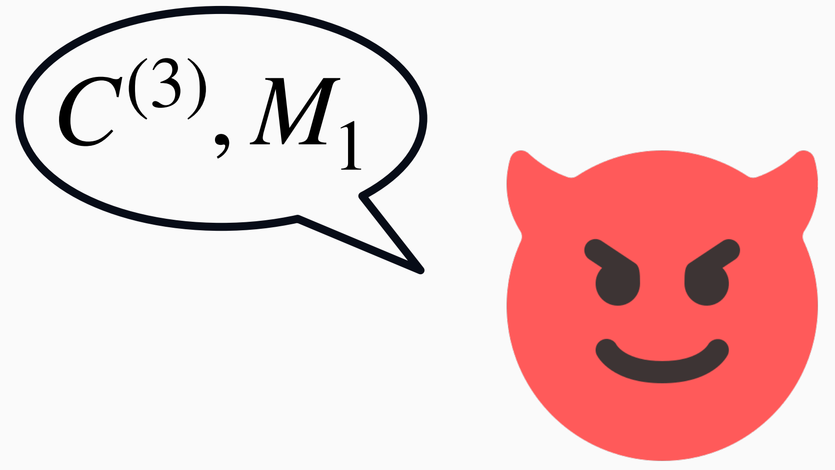

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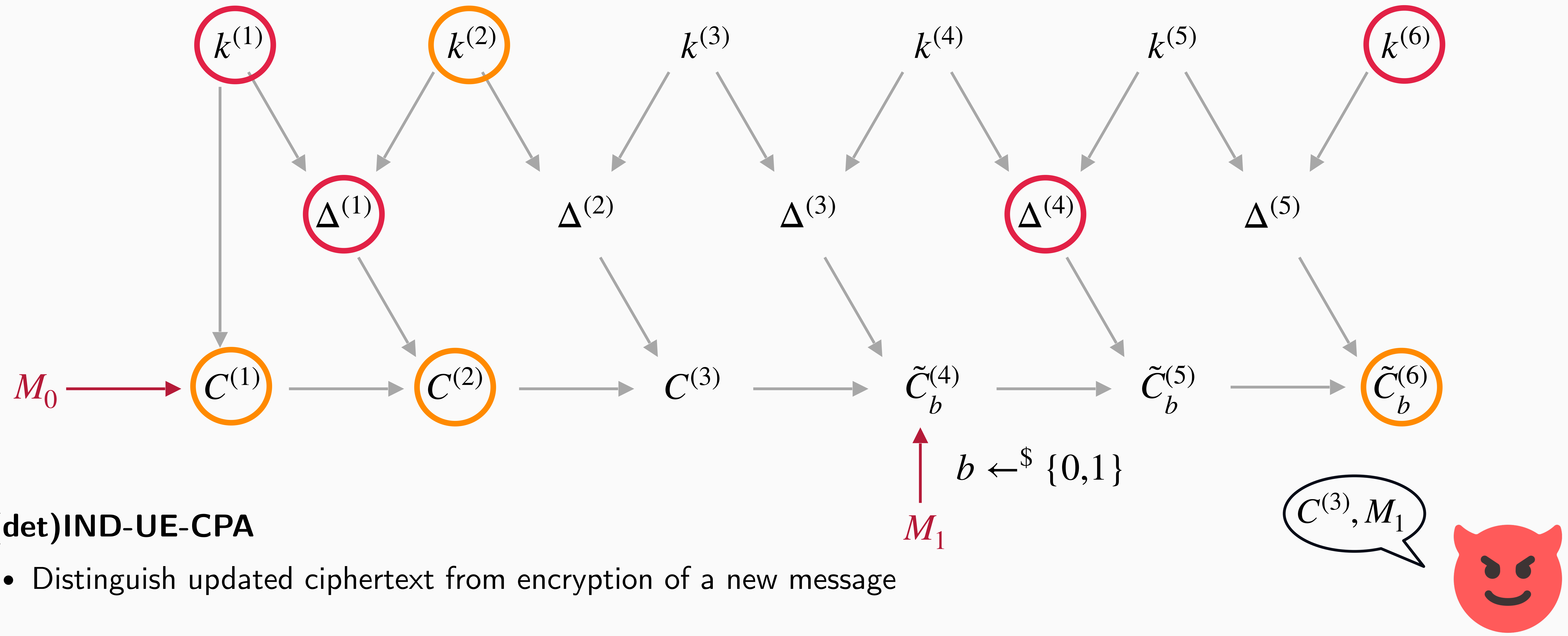

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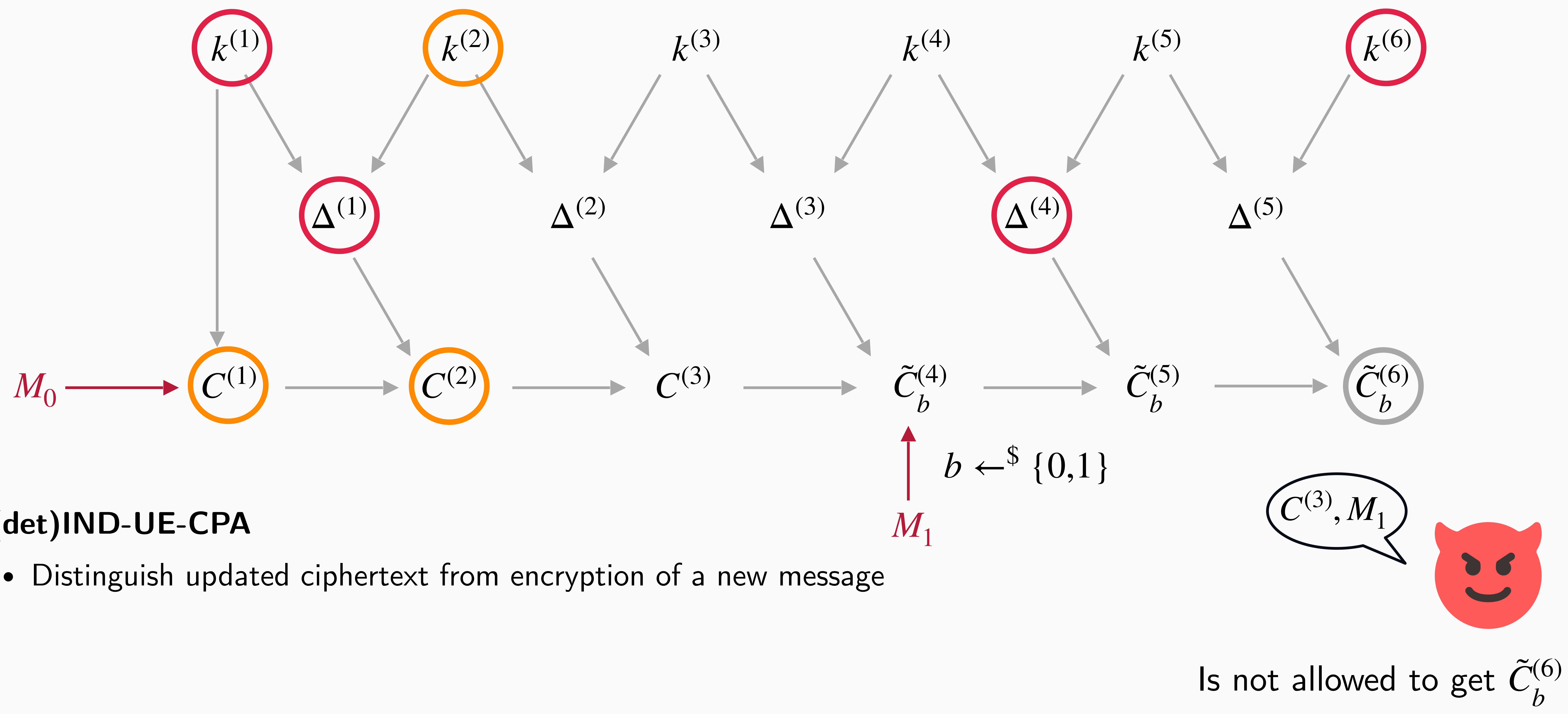

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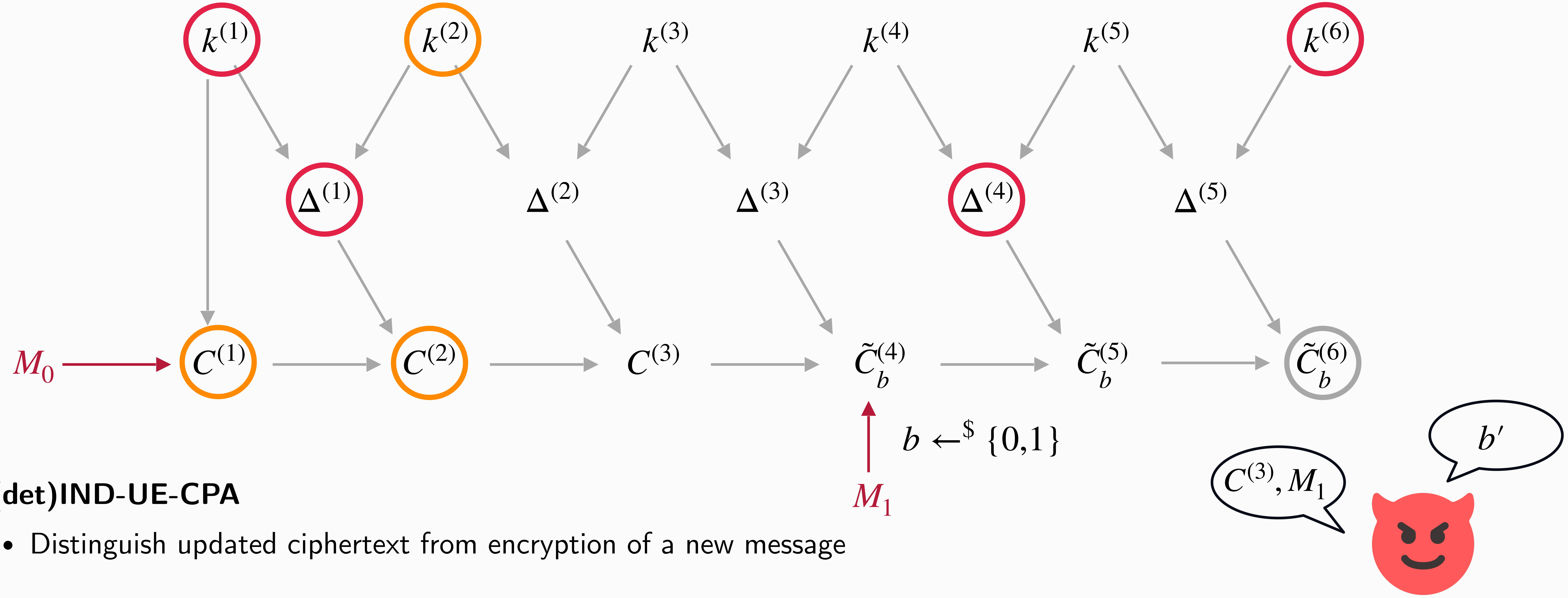

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Is not allowed to get $\tilde{C}_b^{(6)}$

Cryptographic Group Actions

Definition: Group Action

Let $(\mathscr{G}, \cdot\,)$ be a group with identity element e and $\mathscr{X}$ a set. A group action is a map

$$\star : \mathscr{G} \times \mathscr{X} \to \mathscr{X}$$

which satisfies

1. Identity: $e \star x = x$ for all $x \in \mathscr{X}$
2. Compatibility: $(g \cdot h) \star x = g \star (h \star x)$ for all $g, h \in \mathscr{G},\ x \in \mathscr{X}$

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Computational Problems

- DLOG: given $(x, g \star x)$ for $g \xleftarrow{\$} \mathcal{G}$, compute g.
- CDH: given $(x,\ g \star x,\ h \star x)$ for $g, h \xleftarrow{\$} \mathcal{G}$, compute $gh \star x$.
- DDH: given $(x,\ g \star x,\ h \star x,\ z)$, decide whether $z = gh \star x$ or random

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CSIDH [AC:CLMPR18]

$\mathcal{G}$ = isogenies between elliptic curves

$\mathcal{X}$ = supersingular elliptic curves over $\mathbb{F}_p$

Group Action UE Scheme

GAINE [PQCRYPTO:LerRom24]

- Adaptation of SHINE [C:BDGJ20] to group actions
- Key $k \in \mathcal{G}$, ideal cipher $\mathsf{IC} : \{0,1\}^{\ell} \times \{0,1\}^{\lambda} \to \mathcal{X}$ maps message and random nonce to the set ("mappable")

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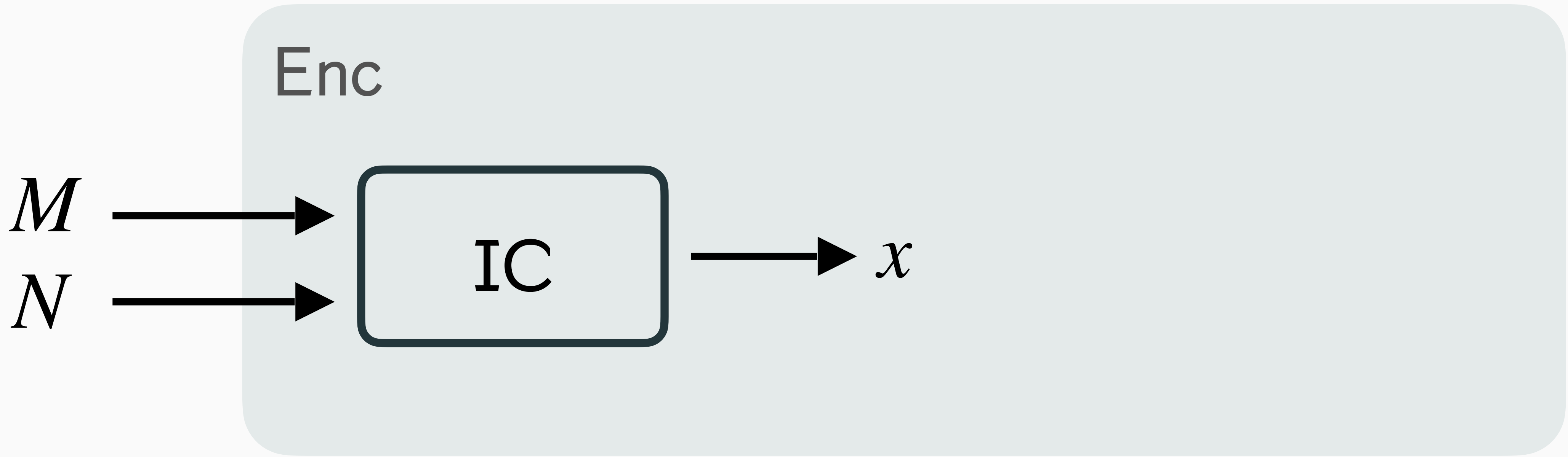

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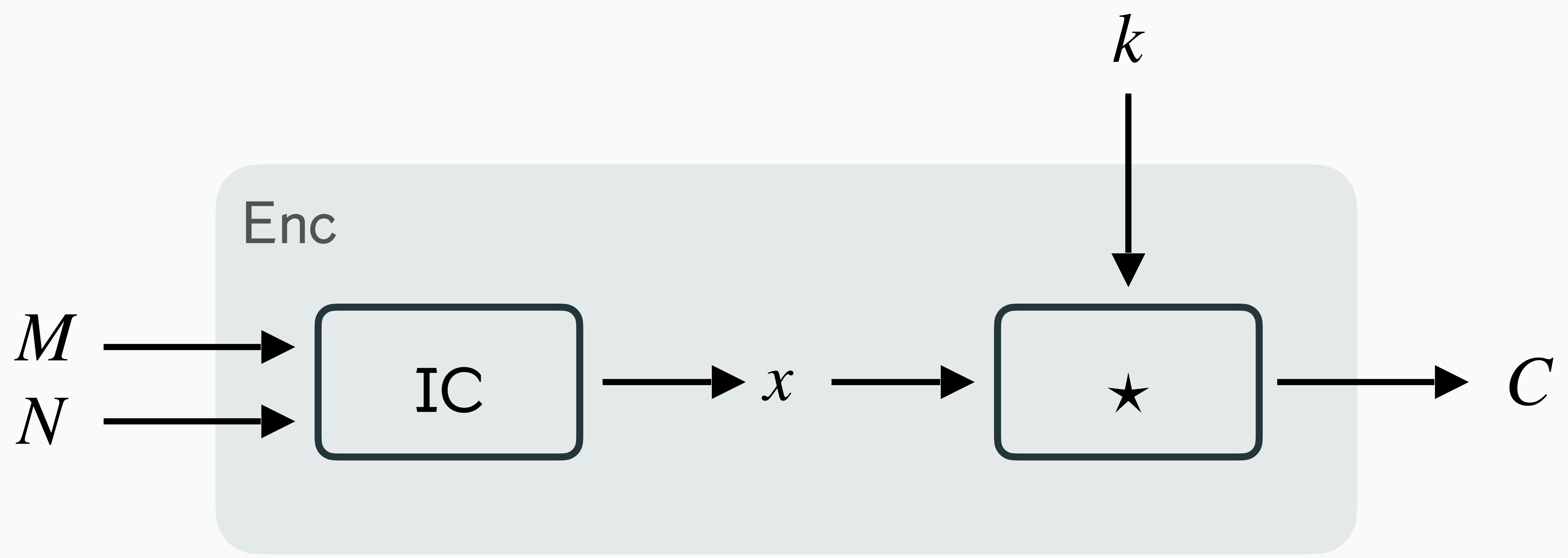

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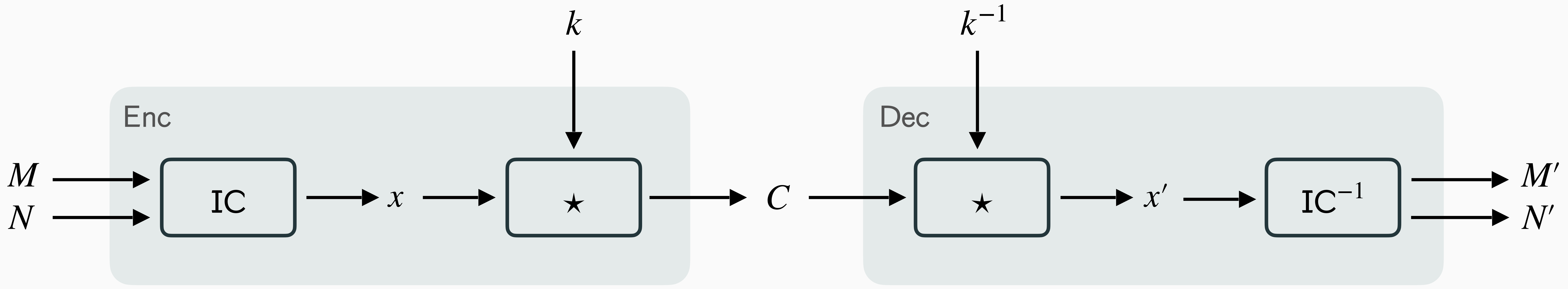

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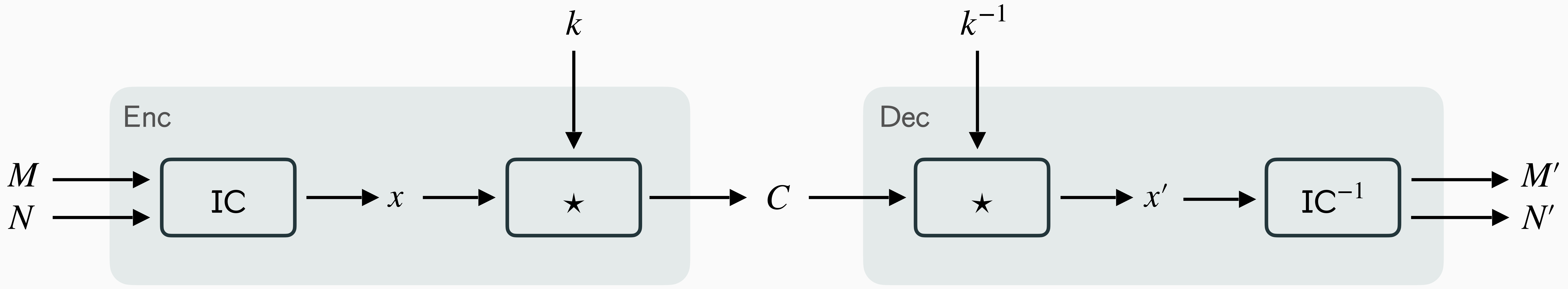

But: For CSIDH we do not know how to map into $\mathcal{X}$ [EPRINT:BBDFGKMPSSTVVWZ22,EPRINT:MulMurPin22].

Our Schemes

Scheme 1: BIN-UE

Message space $\mathscr{M} = \{0,1\}^n \setminus \{0^n, 1^n\}$, $M = (m_1, \ldots, m_n)$ for some $n > 1$

Key space $\mathscr{K} = \mathscr{G}^n$, need some "ordering" for set elements in $\mathscr{X}$

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(can be done by sampling from $\mathscr{G}$)

$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} \star \begin{pmatrix} x_{m_1} \\ x_{m_2} \\ \vdots \\ x_{m_n} \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{pmatrix}$$

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Token Generation and Update

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Decryption

If $|\{x'_0, x'_1, \ldots x'_n\}| = 2$:

Parse bits of M

$$\begin{pmatrix} k_1'^{-1} \\ k_2'^{-1} \\ \vdots \\ k_n'^{-1} \end{pmatrix} \star \begin{pmatrix} C'_1 \\ C'_2 \\ \vdots \\ C'_n \end{pmatrix} = \begin{pmatrix} x'_1 \\ x'_2 \\ \vdots \\ x'_n \end{pmatrix}$$

CPA Security of BIN-UE

Firewall technique [C:BDGJ20]

- Guess start and end of insulated regions (firewalls)
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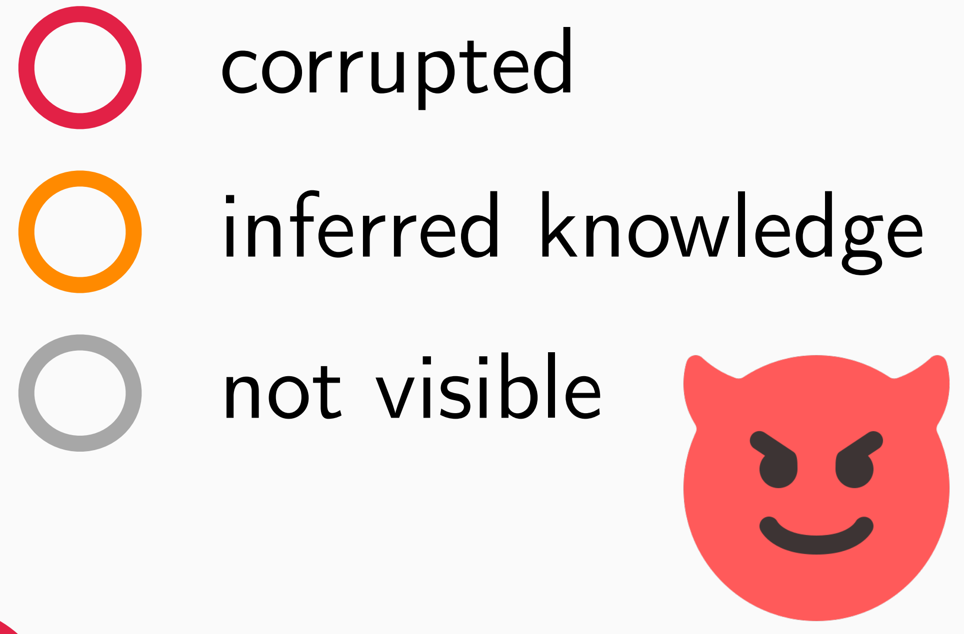

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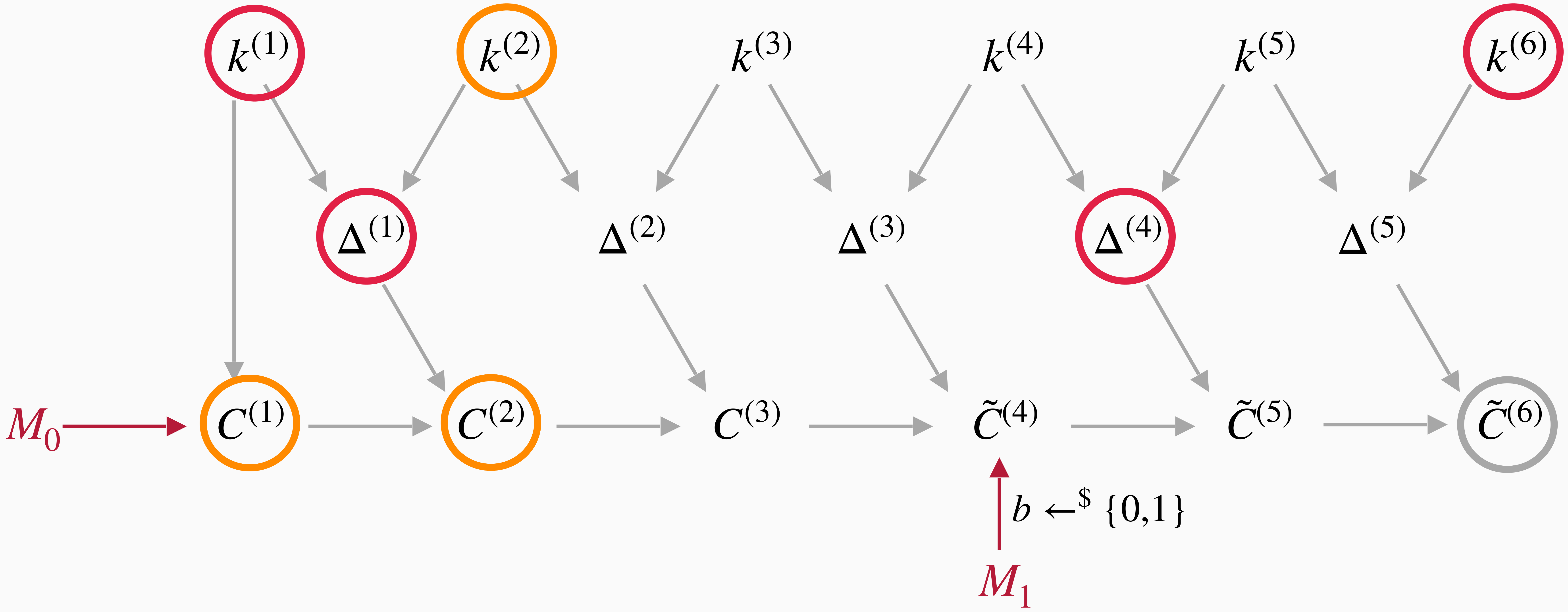

CPA Security of BIN-UE

Firewall technique [C:BDGJ20]

- Guess start and end of insulated regions (firewalls)
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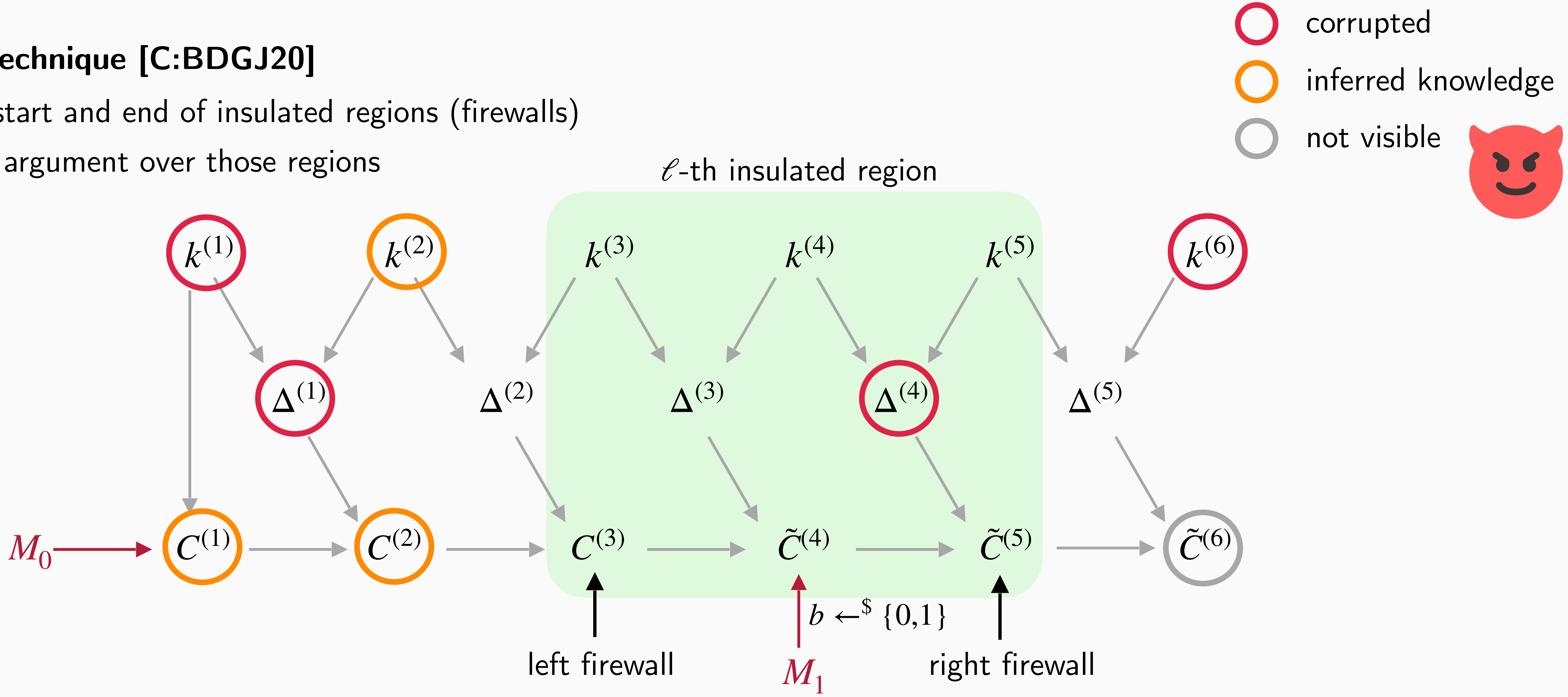
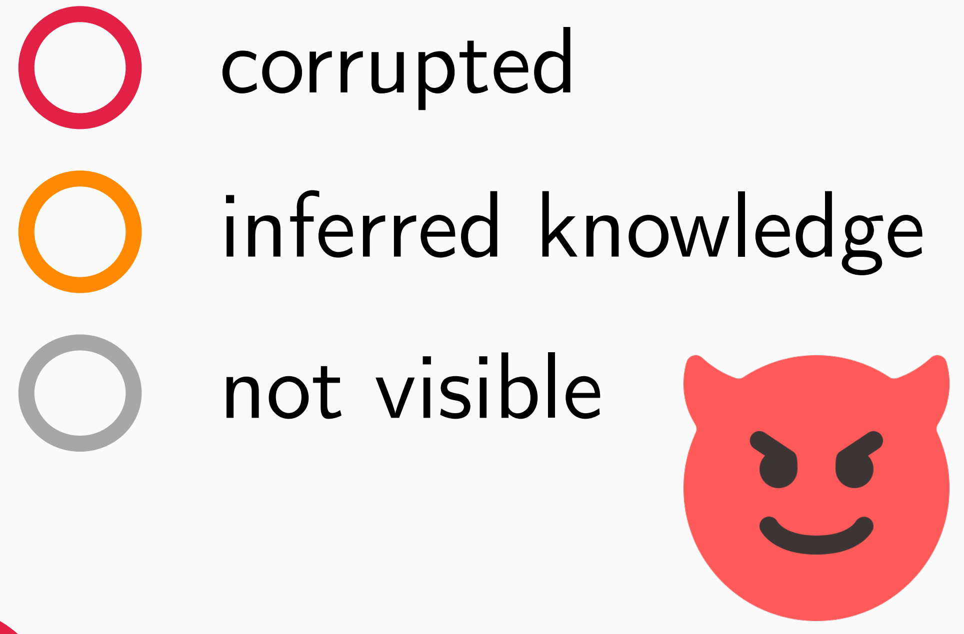


CPA Security of BIN-UE



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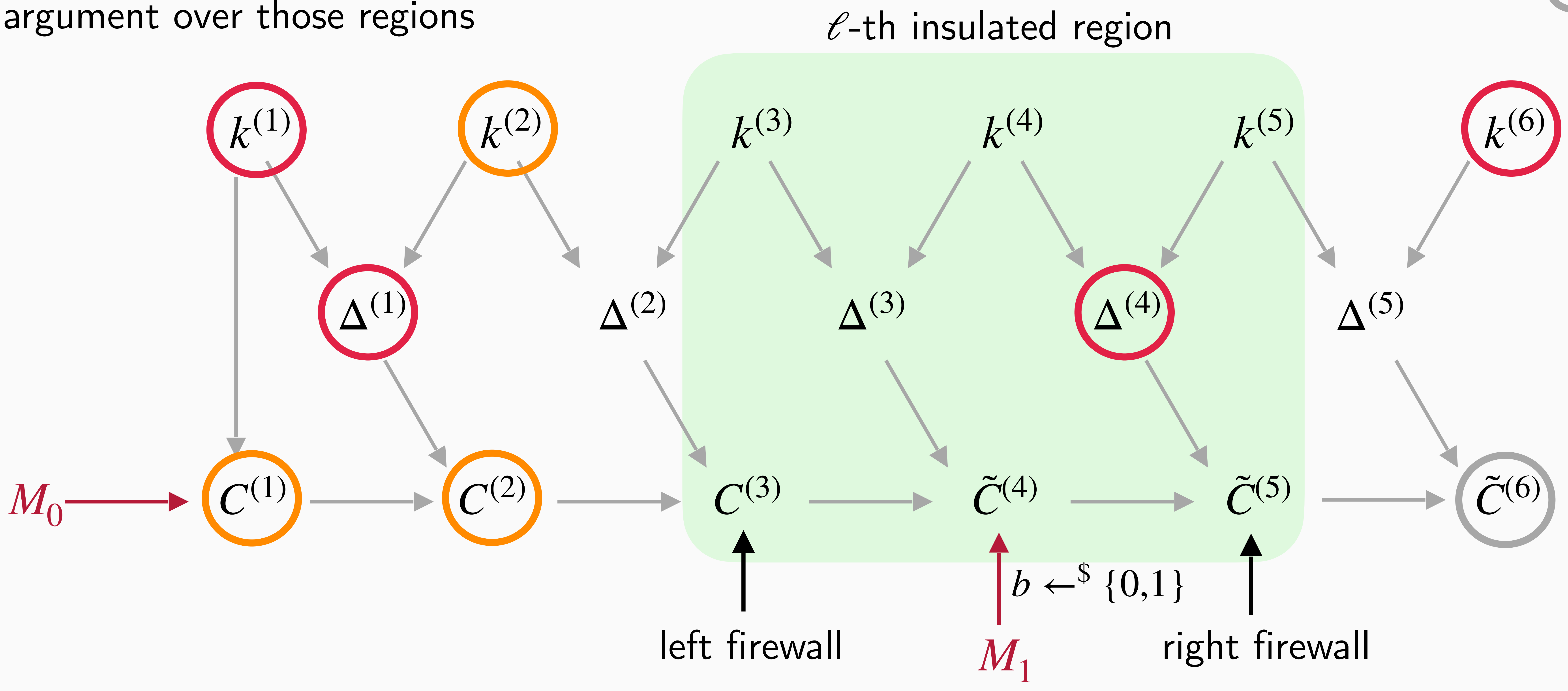

Goal: replace $\tilde{C}_i^{(j)} = k_i^{(j)} \star x_{m_{b,i}}$ inside insulated regions with random elements from $\mathcal{X}$

- Use (multi-instance) group action DDH: given $(x, x_b, k \star x, u \star x_b)$, decide whether $u = k$ or random

Scheme 2: COM-UE

Observations

- BIN-UE (as most other UE schemes) is malleable
- It is randomness-recoverable and randomness-preserving

  $\Rightarrow$ x_0, x_1 are available to an adversary in the security game

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- We define encryption as BIN-UE.Enc$(k, M \| T; r)$, where
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Encryption

$$\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \\ \hline k_{n+1} \\ \vdots \\ k_{n+\ell} \end{pmatrix} \star \begin{pmatrix} x_{m_1} \\ x_{m_2} \\ \vdots \\ x_{m_n} \\ \hline x_{t_1} \\ \vdots \\ x_{t_\ell} \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \\ \hline C_{n+1} \\ \vdots \\ C_{n+\ell} \end{pmatrix}$$

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Ciphertext Integrity

- Should be hard to forge valid ciphertext without knowledge of k
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[C:BDGJ20]: $\underbrace{\textbf{IND-UE-CPA}}_{\text{same as for BIN-UE}} + \textbf{INT-CTXT} \Rightarrow \textbf{IND-UE-CCA}$

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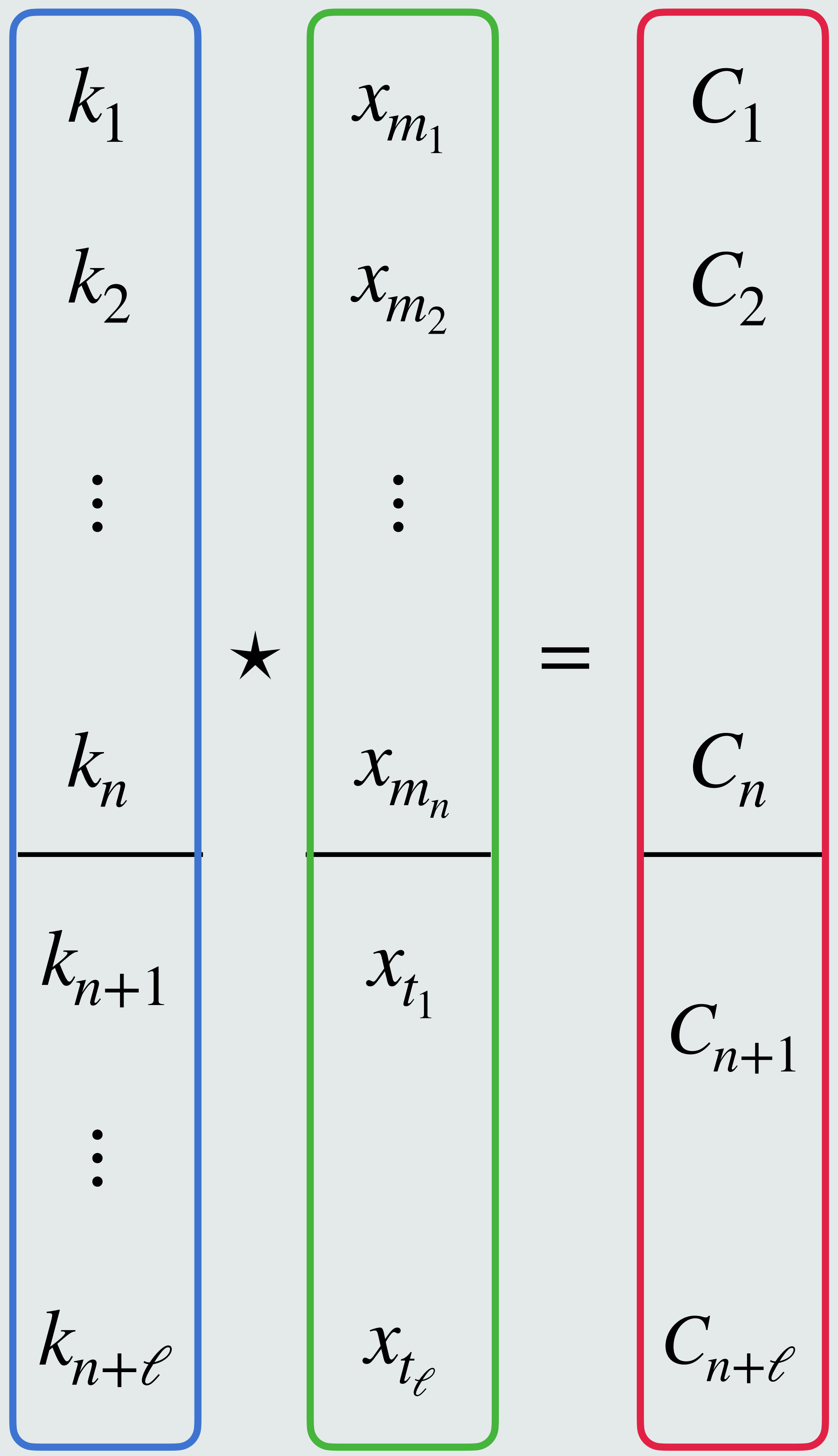

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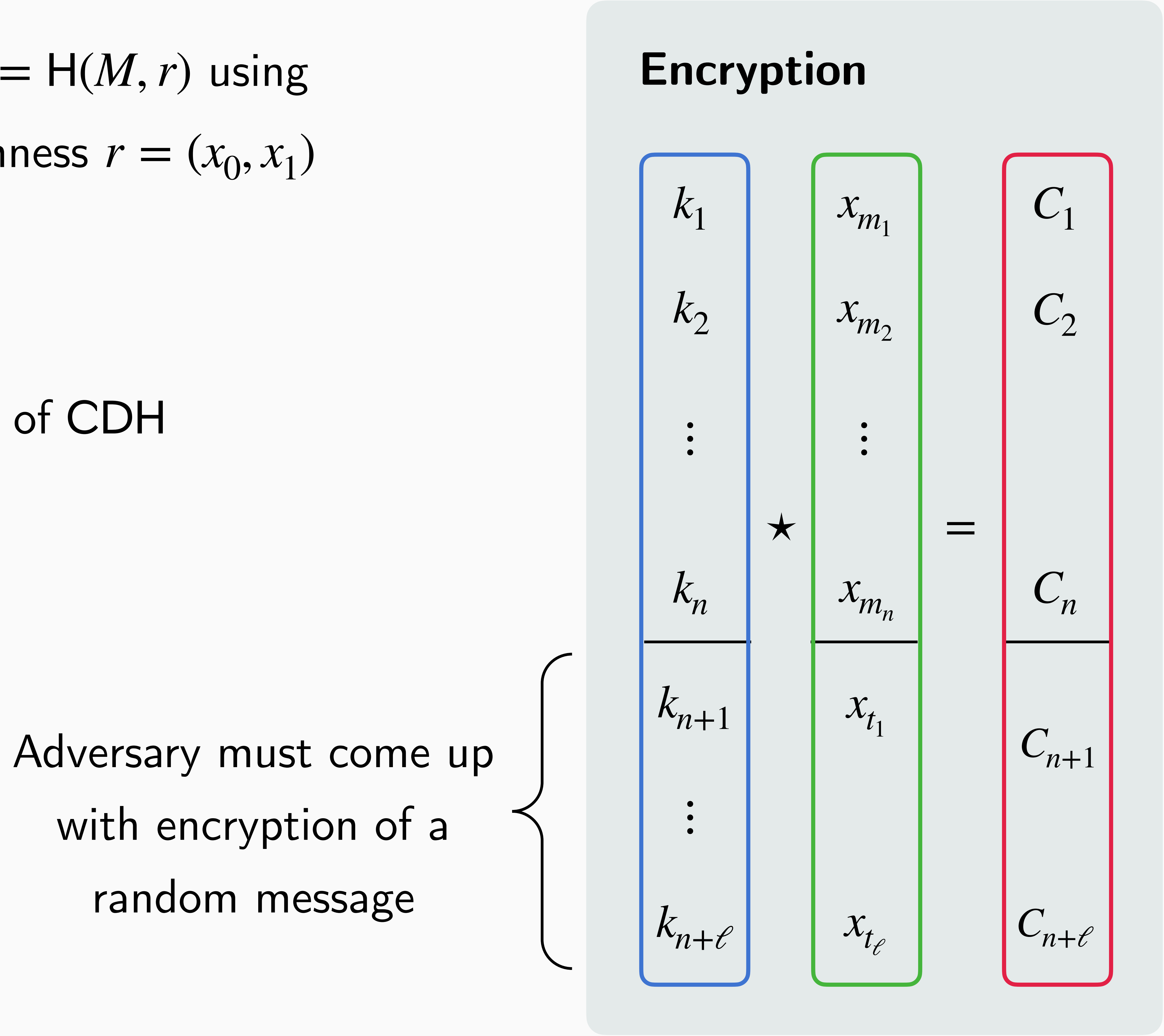
Intuition for INT-CTXT

- Forging a ciphertext allows to solve a non-standard variant of CDH
- Embed the challenge by modeling H as a random oracle



Conclusion

Summary

- Updatable encryption from group actions requires some form of mappability
- Since CSIDH does not allow mapping into the set, we use a bit-wise approach
- BIN-UE achieves CPA security relying on (multi-instance) DDH for group actions
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Thank you!