PROBLEM SHEET 4

4.1 Given $f(x - ct)$, where x and c are constant, show that

$$\frac{d^2}{dt^2} f(x - ct) = c^2 f''(x - ct),$$

and calculate this expression when $f(u) = \sin u$.

4.2 Classify the stationary point of $y = x^{-2}\ln x$, where $x > 0$.

4.3 Classify the stationary points of $y(x) = x^2 - 3x + 2$.

4.4 The numbers x and y are subject to the constraint $x + y = \pi$. Find the values of x and y for which $\cos(x)\sin(y)$ takes its minimum value.

4.5 Sketch the graph of
$$y = \frac{x}{1 + x^2}.$$

4.6 Sketch the graph of
$$y(x) = \tan(2x) \quad \text{for} \quad -\frac{3\pi}{4} \leq x \leq \frac{3\pi}{4}.$$

4.7 Sketch the graph of $y = x\ln x$ for $x > 0$.

4.8 Sketch the graph of
$$y = \frac{x^3}{2x - 1}$$
showing clearly on your sketch any asymptotes.

4.9 Sketch the graph of
$$y = x\cos(3x) \quad \text{for} \quad 0 \leq x \leq 2\pi.$$