PROBLEM SHEET 8

8.1 Obtain the components of the vectors below where L is the magnitude and θ the angle made with the positive direction of the x axis ($-180^\circ < \theta \leq 180^\circ$).
(a) $L = 3, \theta = 60^\circ$;
(b) $L = 3, \theta = -150^\circ$.

8.2 Two ships, S_1 and S_2 set off from the same point Q. Each follows a route given by successive displacement vectors. In axes pointing east and north, S_1 follows the path to B via $\overrightarrow{QA} = (2, 4)$, and $\overrightarrow{AB} = (4, 1)$. S_2 goes to E via $\overrightarrow{QC} = (3, 3)$ $\overrightarrow{CD} = (1, 1)$ and $\overrightarrow{DE} = (2, -3)$. Find the displacement vector $\overrightarrow{BE}$ in component form.

8.3 Sketch a diagram to show that if A, B, C are any three points, then $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$. Formulate a similar result for any number of points.

8.4 Sketch a diagram to show that if A, B, C, D are any four points, then $\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD}$. Formulate a similar result for any number of points.

8.5 Two points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. In terms of $\mathbf{a}$ and $\mathbf{b}$ find the position vectors of the following points on the straight line passing through A and B.
(a) The mid-point C of AB;
(b) a point U between A and B for which $AU/UB = 1/3$.

8.6 Suppose that C has position vector $\mathbf{r}$ and $\mathbf{r} = \lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$ where λ is a parameter, and A, B are points with $\mathbf{a}, \mathbf{b}$ as position vectors. Show that C describes a straight line. Indicate on a diagram the relative positions of A, B, C, when $\lambda < 0, 0 < \lambda < 1$, and $\lambda > 1$.

8.7 Find the shortest distance from the origin of the line given in vector parametric form by $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where

$$\mathbf{a} = (1, 2, 3) \quad \text{and} \quad \mathbf{b} = (1, 1, 1)\,,$$

and t is the parameter (Hint: use a calculus method, with t as the independent variable.)

8.8 $ABCD$ is any quadrilateral in three dimensions. Prove that if P, Q, R, S are the mid-points of AB, BC, CD, DA respectively, then $PQRS$ is a parallelogram.

8.9 ABC is a triangle, and P, Q, R are the mid-points of the respective sides BC, CA, AB. Prove that the medians AP, BQ, CR meet at a single point G (called the centroid of ABC; it is the centre of mass of a uniform triangular plate.)

8.10 Show that the vectors $\overline{0A} = (1, 1, 2), \overline{0B} = (1, 1, 1)$, and $\overline{0C} = (5, 5, 7)$ all lie in one plane.