MATHEMATICS ADMISSIONS TEST

For candidates applying for Mathematics, Computer Science, or one of their joint degrees at the UNIVERSITY OF OXFORD.

October 2024

Time allowed: $2\frac{1}{2}$ hours

In 2024, the MAT was administered by Pearson VUE, and candidates answered the questions electronically. There were 25 multiple-choice questions and two longer questions with multiple parts, for which candidates typed answers.

The multiple-choice questions (Q1 – Q25) will not be released.
The long questions (Q26 and Q27) are reproduced below.

Question 26

In this question we will write (x, y) for a vector instead of the usual $\begin{pmatrix} x \\ y \end{pmatrix}$ notation. So, for example, $3(2, 1) = (6, 3)$.

This question is about vectors (x, y) where x and y are whole numbers, and whether or not such vectors can be written in the form $a(5, 0) + b(0, 7) + c(2, 1)$ where a and b and c are whole numbers each greater than or equal to zero.

We will consider the set S of vectors (p, q) with $0 \leq p \leq 4$ and $0 \leq q \leq 6$, with p and q whole numbers.

Then, by considering the vectors (x, y), $(x, y) - (2, 1)$, $(x, y) - 2(2, 1)$, $\ldots$, $(x, y) - 34(2, 1)$, we will find conditions on x and y that imply that (x, y) can be written in the form $a(5, 0) + b(0, 7) + c(2, 1)$ where a and b and c are whole numbers each greater than or equal to zero.

(i) Consider the set S of vectors (p, q) with $0 \leq p \leq 4$ and $0 \leq q \leq 6$, with p and q whole numbers.

  (a) How many vectors are in S?

  (b) Explain why for any vector (x, y) with x and y whole numbers, we can find whole numbers a and b, and a vector $\mathbf{v}$ in S such that

$$(x, y) = a(5, 0) + b(0, 7) + \mathbf{v}.$$

In the rest of this question, we'll call such a vector $\mathbf{v}$ the *residue* of (x, y), and we will assume that the residue is uniquely defined for each vector (x, y).

3 marks

(ii) Consider vectors $(x, y) + k(2, 1)$ and $(x, y) + m(2, 1)$ where k and m are whole numbers.

  (a) Prove that if these vectors have the same residue (as defined in the previous part of the question), then $(k - m)$ is a multiple of 35.

  (b) Explain why the vectors (x, y), $(x, y) - (2, 1)$, $(x, y) - 2(2, 1)$, $\ldots$, $(x, y) - 34(2, 1)$ all have different residues.

4 marks

(iii) Hence show that if x is at least 68 and y is at least 34, with x and y whole numbers, then the vector (x, y) can be written in the form $a(5, 0) + b(0, 7) + c(2, 1)$ where a and b and c are whole numbers each greater than or equal to zero.

You do not need to find the values of a and b and c.

4 marks

(iv) A student claims: "if x and y are whole numbers with $x > 0$ and $y > 0$ and $x + y$ at least 102 then (x, y) can be written in the form $a(5, 0) + b(0, 7) + c(2, 1)$ where a and b and c are whole numbers each greater than or equal to zero."

Is the student's claim correct? Justify your answer.

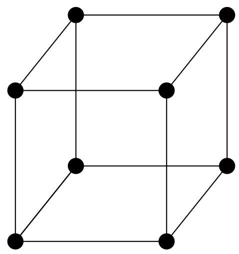
4 marks

Question 27

The faces of a cube are each painted either red or blue.

An ant is positioned on each of the eight corners of the cube, and each ant can only see the three faces that meet at that corner of the cube.

In this question, the ants will be asked about the faces that they can see, and the ants will always answer truthfully.



(i) The ants are each asked "Can you see an even number of red faces?". Each ant answers either yes or no. Explain why the number of ants that say yes is even.

3 marks

(ii) Is it possible that all eight of the ants can each see exactly two red faces? Justify your answer.

2 marks

(iii) The ants are each asked "Can you see at least one red face?". Explain why it is impossible for exactly five of the ants to say yes and exactly three to say no.

3 marks

(iv) Suppose that the four ants on the corners of the top face of the cube can see exactly 0, 1, 1, and 2 red faces each, in some order. How many blue faces might there be in total? Find all possibilities, and explain your answer.

3 marks

(v) A three-dimensional shape is constructed such that each face is either a square or a hexagon, with two faces meeting at each edge and three faces meeting at each corner. Each face of the shape is painted either red or blue. Consider the edges where a red face meets a blue face. Explain why the number of such edges is even.

4 marks