SOLUTIONS FOR ADMISSIONS TEST IN
MATHEMATICS, COMPUTER SCIENCE AND JOINT SCHOOLS
OCTOBER 2024

Question 26

(i) (a) 35 (five choices for p, 7 choices for q, with the choices made independently).

(b) Take the remainder of the x-component after dividing by 5 and the remainder of the y-component dividing by 7.

3 marks

(ii) (a) Suppose $(x,y)+k(2,1)$ and $(x,y)+m(2,1)$ have the same residue $\mathbf{v}$. Then

$$(x,y)+k(2,1)=a(5,0)+b(0,7)+\mathbf{v}$$

and

$$(x,y)+m(2,1)=c(5,0)+d(0,7)+\mathbf{v}$$

for some a, b, c, d positive whole numbers. Now take the difference between these two equations for $(k-m)(2,1)=(a-c)(5,0)+(b-d)(0,7)$.

So $(k-m)$ is a multiple of 5 (from the x-component) and a multiple of 7 (from the y-component). So it's a multiple of 35.

(b) These are vectors of the above form with $k=0$, $k=-1$, $\ldots$, $k=-34$. No two numbers of that set differ by as much as 35. So no two of them can have the same residue.

4 marks

(iii) Consider the vectors (x,y), $(x,y)-(2,1)$, $\ldots$, $(x,y)-34(2,1)$ as in the previous part. By the previous part, these all have different residues, and there are 35 of them, so one of them has the residue $(0,0)$.

That one has x-component a multiple of 5 and y-component a multiple of 7. Moreover, the components are positive or zero because $x>67$ and $y>33$ so $x-68$ and $y-34$ are positive or zero.

So that vector can be written as $a(5,0)+b(0,7)$.

Then x is $a(5,0)+b(0,7)+c(2,1)$ where c is the coefficient of $(2,1)$ which we subtracted to find something with residue $(0,0)$. **4 marks**

(iv) No. Consider for example $(100,2)$. Looking at the y-component, we must have $b=0$ and $c=2$ to make a y-component of 2. But then from the x-component we would have $5a+4=100$ which doesn't have a whole-number solution for a.

4 marks

Question 27

(i) Each face is seen by 4 ants. If we add up how many red faces each ant can see, we should get four times the number of red faces, giving us an even number in total.

If an odd number of the ants say no to this question, then the total number of red faces those ants can see is odd. The ants that say yes can each see an even number of red faces. So the total number of red faces would be odd. That's not OK, by the logic above.

Alternatively, list the ten possibilities for how the cube may be painted (up to rotation), and check how many ants can see an even number of red faces in each case. **3 marks**

(ii) Yes, this is possible if exactly two of the face are blue, and the other four are red, with the two blue faces opposite each other.

Do not accept arguments that just mention that this would be consistent with the previous part. **2 marks**

(iii) If exactly five ants can see a red face, then there is at least one red face somewhere. Four ants can see that red face. Then because five is larger than four, there must be at least one more red face. But that will be seen by either two more ants (if the face is adjacent to the first red face) or four more ants (if it is opposite). Then there might be even more red faces, which cannot reduce the number of ants who can see at least one red face. So it's impossible for exactly five red ants to each see at least one red face.

Do not accept "the number that say yes must be even", because it could be seven (if exactly three faces are red, meeting at a corner). **3 marks**

(iv) The 0 means that this top face is blue. Then the 2 means that two of the "side" faces are red. The other two side faces must be blue, else another ant would have said 2. That's all the faces except the bottom face. We can't deduce the colour of that face, because switching the colour of that face wouldn't change anything that the four ants around the top face can see. So there are either three or four blue faces in total. **3 marks**

(v) Place a beetle on each edge (so that it can only see the two faces that meet at that edge) and ask each beetle to count how many red faces it can see. Consider the total of the beetle's individual counts. Each red face increases the total by four or six, so the total must be even. Each beetle can see either 0 or 1 or 2 red faces. Since 0 and 2 are even, the number of beetles that can see exactly one red face must be even too, for a even total. Those correspond precisely to the edges where a red face meets a blue face.

Alternatively, consider the number of faces painted red. If there are no red faces, then the number of red-blue edges is zero. Now consider a general painting and change a face from blue to red. This face meets B blue faces and R faces say, with $B + R$ even (either 4 or 6). The number of red-blue edges was previously R and now it's B, a change of $B - R$. But since $B + R$ is even, $B + R - 2R$ is also even. So the change is even.

Do not accept arguments that assume the shape is a cube, or made of only squares. It could be a truncated octahedron! **4 marks**