PRACTICE MAT SOLUTIONS

These solutions are for the practice test on the Pearson VUE system, created ahead of MAT 2024. The questions are all past MAT questions from 2007–2022. The answers are collated below.

1. The centre is at $(3, 4)$ and one corner is at $(1, 5)$, which is a displacement of $(-2, 1)$. There's another corner opposite, at $(3, 4) - (-2, 1) = (5, 3)$, but this is not one of the options. To find the other corners, we need to find a vector with equal magnitude, but at right-angles to $(-2, 1)$. Rotating by $90°$ gives $(1, 2)$. The other corners are at $(3, 4) \pm (1, 2)$, which includes $(2, 2)$.
   The answer is (2,2).

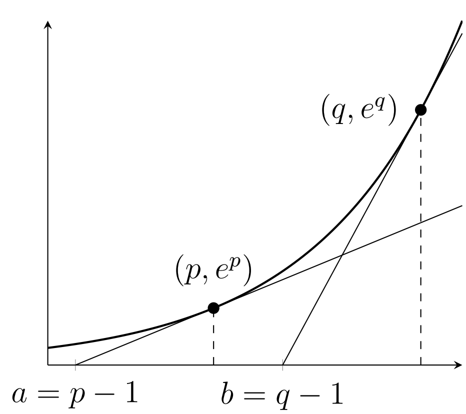
2. Using the difference of two squares, this is $\int_0^1 e^{2x} - x^2 \, \mathrm{d}x$. Notice that, since the derivative of e^{2x} is $2e^{2x}$, this integral is
   $$\left[\frac{1}{2}e^{2x} - \frac{x^3}{3}\right]_0^1 = \frac{e^2}{2} - \frac{1}{3} - \frac{1}{2} = \frac{3e^2 - 5}{6}$$
   The answer is $\frac{3e^2 - 5}{6}$.

3. Pair the terms to get $-3 - 7 - \cdots - 199$. This is the sum of an arithmetic progression and is equal to $(-3 - 199) \times 50/2 = -5050$
   The answer is -5050.

4. Connect each point to the centre of the circle to split the shape into 12 isosceles triangles, each with angle $30°$ at the centre. Then the area of each triangle is $\frac{1}{2} \times 1 \times 1 \times \sin 30° = \frac{1}{4}$, and there are 12 triangles, for a total area of 3.
   The answer is 3.

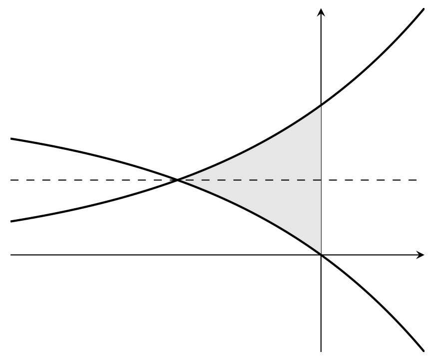
5. The integral is
   $$\int_0^a \sqrt{x} + x^2 \, \mathrm{d}x = \int_0^a x^{1/2} + x^2 \, \mathrm{d}x = \left[\frac{2}{3}x^{3/2} + \frac{1}{3}x^3\right]_0^a = \frac{2}{3}a^{3/2} + \frac{1}{3}a^3.$$
   So we have $2a^{3/2} + a^3 = 15$. This factorises as $(a^{3/2} - 3)(a^{3/2} + 5) = 0$. Since $a > 0$ we want $a^{3/2} > 0$, so it's 3, so $a = 3^{2/3}$.
   The answer is $a = 3^{2/3}$.

6. The gradient at p is e^p and so the tangent is $y = e^p(x - p) + e^p$. This crosses the x-axis when $e^p(a-p)+e^p = 0$ which happens if $a = p-1$. Similarly $q = 1-b$ so $p-a = q-b$ (they're both 1).



The answer is $p - a = q - b$.

7. The intersection point is at $e^x = 1 - e^x$ which happens when $2e^x = 1$, that is $x = -\ln 2 < 0$.



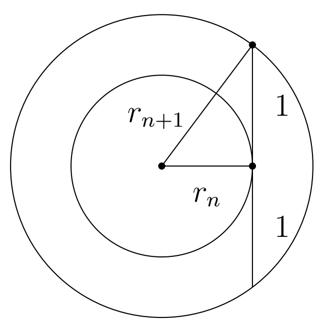
The area has reflectional symmetry in the line $y = \frac{1}{2}$ so we want

$$2\int_{-\ln 2}^{0} e^x - \frac{1}{2}\,\mathrm{d}x = 2\left[e^x - \frac{x}{2}\right]_{-\ln 2}^{0} = 2\left((1) - \left(\frac{1}{2} + \frac{\ln 2}{2}\right)\right) = 1 - \ln 2$$

The answer is $1 - \ln 2$.

8. Either $x \geq 0$ and $x^2 + 1 = 3x$ or $x < 0$ and $-x^2 + 1 = -3x$. The first equation has solutions $\frac{1}{2}\left(3 \pm \sqrt{5}\right)$ which are both positive. The second equation has solutions $\frac{1}{2}\left(3 \pm \sqrt{13}\right)$ and only one of these is actually negative. So there are three solutions in total.
The answer is 3.

9. The relationship between circle C_n of radius r_n and circle C_{n+1} of radius r_{n+1} is shown below.

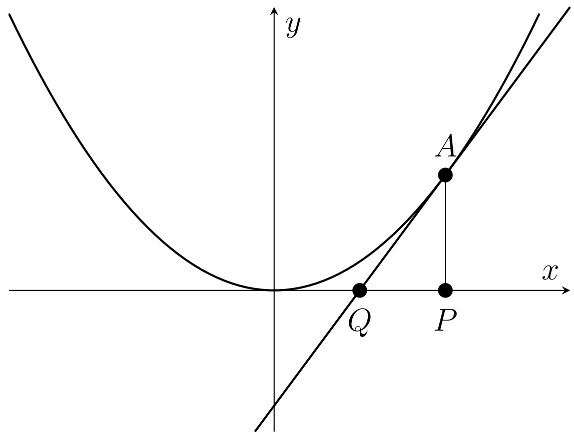


The tangent is at right-angles to the radius, so there's a right-angled triangle with hypotenuse r_{n+1} and other sides r_n and 1. Pythagoras gives $r_{n+1}^2 = r_n^2 + 1$. Since $r_1^2 = 1$, we have $r_2^2 = 2$ and $r_3^2 = 3$ and so on, up to $r_{100}^2 = 100$, so the radius of C_{100} is 10.
The answer is 10.

10. Complete the square for x and for y $(x - 2k)^2 - 4k^2 + (y - 2)^2 - 4 + 8 = k^3 - k$ This is the equation of a circle with centre $(2k, 2)$ and $r^2 = k^3 + 4k^2 - k - 4$ provided that expression is positive. Factorising the cubic as $(k + 4)(k - 1)(k + 1)$ reveals that this happens if and only if either $-4 < k < -1$ or if $k > 1$.
The answer is "if and only if either $-4 < k < -1$ or $k > 1$".

11. $3\cos^2 x + 2\sin x + 1 = 3(1 - \sin^2 x) + 2\sin x + 1 = -3(\sin x - 1/3)^2 + 1/3 + 4$. This takes its maximum value when $\sin x = 1/3$, and that value is $4 + 1/3 = 13/3$.
The answer is $\dfrac{13}{3}$.

12. The tangent is $y = 2ax - a^2$, which crosses the x-axis at $(a/2, 0)$.



The area is $\int_0^a x^2 \, dx$ − (area of APQ) where A is (a, a^2), P is $(a, 0)$ and Q is $(a/2, 0)$. This is $\frac{a^3}{3} - \frac{a^3}{4} = \frac{a^3}{12}$
The answer is $\frac{a^3}{12}$.

13. Factorise the numbers from 1 to 10; this gives

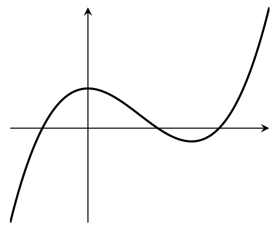
$$\log_{10}(2 \times 5 \times 3^2 \times 2^3 \times 7 \times 2 \times 3 \times 5 \times 2^2 \times 3 \times 2) = \log_{10}(2^8 3^4 5^2 7).$$

Use $\log_{10}(2^2 5^2) = 2$. Use $2^4 3^4 = 6^4$. This gives $2 + \log_{10}(2^2 6^4 7) = 2 + 2\log_{10} 2 + 4\log_{10} 6 + \log_{10} 7$.
The answer is $2 + 2\log_{10} 2 + 4\log_{10} 6 + \log_{10} 7$.

14. The turning points have $\frac{dy}{dx} = 0$ at $x = 1$ and at $x = 3$, so $3 + 2a + b = 0$ and $27 + 6a + b = 0$. Solve these for $a = -6$, $b = 9$. Now $y = 2$ at $x = 1$ so $c = -2$. Then $y = -2$ at $x = 3$ gives $d = -2$.
The answer is -2.

15. In order to make the vector $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$ we would need $a\begin{pmatrix} 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$ where a is the number of times we pick $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and b is the number of times we pick $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.
Since we have six vectors, $a + b = 6$. Solving the simultaneous equations $a + 3b = 10$ and $a + 2b = 8$, we get $a = 4$ and $b = 2$, and we can check that $a + b = 6$ for this solution! So we want exactly two of the six vectors to be $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. There are ${}^6C_2 = 15$ ways that this could happen, each with probability $\frac{1}{64}$, so the answer is $\frac{15}{64}$.
The answer is $\frac{15}{64}$.

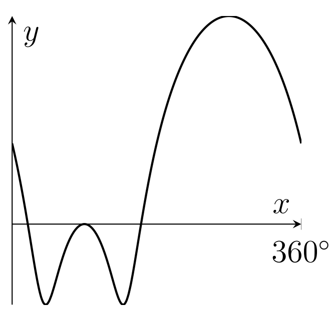
16. The tangent at a is $y = (3a^2 - 3)(x - a) + (a^3 - 3a)$, which passes through $(2, 0)$ if and only if $0 = (3a^2 - 3)(2 - a) + (a^3 - 3a)$. This simplifies to $2a^3 - 6a^2 + 6 = 0$. The left-hand side is a cubic in a and we'd like to know how many roots it has.



The turning points of $2a^3 - 6a^2 + 6$ are at $a = 0$ and $a = 2$, where the value of the cubic is 6 and -2 respectively. So this cubic starts negative, rises to a positive local maximum, then decreases to a negative local minimum before rising again. There are therefore three roots for this cubic, so three values of a for which the tangent to the original cubic passes through the point $(2, 0)$.
The answer is "three values of a".

17. We can use the fact that $\sin^2(90° - n) = \cos^2(n)$ for any n, and $\sin^2 x + \cos^2 x = 1$. So we have $\sin^2 1° + \sin^2 89° = 1$ and $\sin^2 2° + \sin^2 88° = 1$ and so on up to $\sin^2 44° + \sin^2 46° = 1$. We also have $\sin^2 45° = \frac{1}{2}$ and $\sin^2 90° = 1$ for a total of $45\frac{1}{2}$.
The answer is $45\frac{1}{2}$.

18. The function inside the brackets is $6\sin^2 x - 8\sin x + 3$ which is a quadratic for $\sin x$. We could therefore consider the quadratic $6u^2 - 8u + 3$ for $-1 \le u \le 1$. Complete the square to write this as $6\left(u - \frac{2}{3}\right)^2 + \frac{1}{3}$. This reaches a minimum value when $u = \frac{2}{3}$. For $u = \sin x$ in the range $0 \le x \le 360°$, this happens for two values of x both in $0 < x < 180°$. The value there is $\log_2\left(\frac{1}{3}\right) < 0$. Only one of the graphs reaches a negative minimum value twice in that range.
The answer is this graph;



19. $a_1 = 8 \times (3)^4$ and $a_2 = 8(8 \times 3^4)^4 = 8 \times 8^4 \times (3^4)^4$ and $a_3 = 8 \times (8 \times 8^4 \times (3^4)^4)^4$. In a similar way,

$$a_n = 8 \times 8^4 \times 8^{16} \times \cdots \times 8^{4^{n-1}} \times 3^{(4^n)}$$

The exponent on the 8s is $1 + 4 + 16 + \cdots + 4^{n-1} = \frac{1}{3}(4^n - 1)$. Then $8^{1/3} = 2$. Also use $4^n = 2^{2n}$.

$$a_n = 2^{2^{2n}-1} 3^{(2^{2n})} = \frac{6^{2^{2n}}}{2}$$

So for $n = 10$ we get $\dfrac{6^{(2^{20})}}{2}$.

The answer is $\dfrac{6^{(2^{20})}}{2}$.

20. We'll calculate the square of $(x+1+x^{-1})$ first;

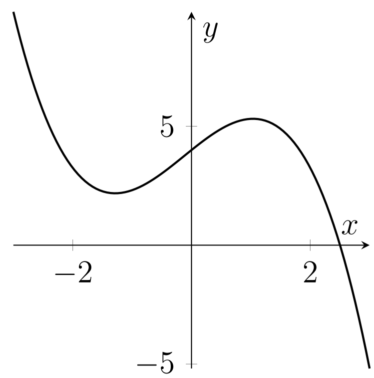
$$x^2+2x+3+2x^{-1}+x^{-2}$$

(I drew a square grid to keep track of all the cross-terms, and there's a nice pattern which helps). Now if we were to square this expression, the constant term independent of x would be

$$2(x^2)(x^{-2})+2(2x)(2x^{-1})+3^2$$

Most of the terms have a factor of 2 because they occur in either order. This sum is $2+8+9=19$. The answer is 19.

21. One of the options looks like a quadratic, three of the options look like cubics, and one of the options looks like a quartic (a polynomial of degree 4), so we might expect that one of the cubics is the derivative of the quartic, and the quadratic is the derivative of another of the cubics, leaving one remaining cubic that could be $h(x)$. Checking the sign and the locations of the zeros of the possible derivatives leaves one function over as a possible candidate for $h(x)$;



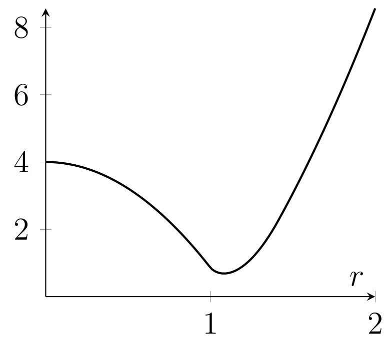
We should check that it's definitely not the derivative of any of the other options, and that its derivative is not any of the other options. Note that it's only got one zero, and it's negative for large x. This is enough to see that it's not the derivative of any of the other functions. It's also not the case that the derivative of this graph is any of the other graphs; such a graph would have two zeros for the two turning points, but only of the functions has two zeros, and the sign of that graph is wrong (positive where the gradient of this graph is negative). So we conclude that the graph above is the "odd one out".
The answer is the graph above.

22. The geometric sum converges if $\frac{1}{|\tan x|}<1$, and converges to

$$\frac{1}{\tan x}+\frac{1}{\tan^2 x}+\frac{1}{\tan^3 x}+\cdots=\frac{\frac{1}{\tan x}}{1-\frac{1}{\tan x}}=\frac{1}{\tan x-1}.$$

Let $u=\tan x$, then this sum is equal to $\tan x$ if and only if $u^2-u=1$, so if and only if $u=\frac{1}{2}(1\pm\sqrt{5})$. But the condition $|\tan x|>1$ means that we must take $u=\frac{1}{2}(1+\sqrt{5})$. There is one value of x in the range $-90°<x<90°$ with this value of $\tan x$.
The answer is 1.

23. For $r < 1$, $A = 0$ and $B = 4 - \pi r^2$. For $r > \sqrt{2}$, $A = \pi r^2 - 4$ and $B = 0$, which eliminates two of the options. Now two of the remaining options have $A + B = 0$ at some value of r, which would require $A = 0$ and $B = 0$, that is; no area inside the square that's not also inside the circle and vice versa. This is impossible. There is only one remaining option.
The answer is this graph



24. Let's call the product of the first n terms b_n. Then we have $b_n = a_n b_{n-1}$ (that's how the product works). We also have the definition of a_n to interpret; it's one more than the previous product, so $a_n = b_{n-1} + 1$. We can use this to eliminate b_n and b_{n-1} from the previous equation, to get $a_{n+1} - 1 = a_n(a_n - 1)$. Adjust the subscripts and rearrange for $a_n = a_{n-1}(a_{n-1} - 1) + 1$.
The answer is $a_n = a_{n-1}\,(a_{n-1} - 1) + 1$.

25. We must have $|AB| = |BC|$ so $\sqrt{(b-a)^2 + (c-b)^2} = \sqrt{(c-b)^2 + (d-c)^2}$.

We must also have $|BC| = |CD|$ so $\sqrt{(c-b)^2 + (d-c)^2} = \sqrt{(d-c)^2 + (a-d)^2}$.

These conditions are equivalent to $(b-a)^2 = (d-c)^2$ and $(c-b)^2 = (a-d)^2$ respectively.

Using the difference of two squares, the first is equivalent to $(a-b+c-d)(a-b-c+d) = 0$ and the second is equivalent to $(a-b+c-d)(a+b-c-d) = 0$.

In each case, we can't have both brackets equal to zero because $c \neq d$ and $b \neq c$ because the numbers are distinct. So either $a-b+c-d = 0$ or both of $a-b-c+d = 0$ and $a+b-c-d = 0$. That second case would imply that $a-c = 0$, but the numbers are distinct so that's impossible. So we're left with just the case that $a-b+c-d = 0$. We can also check that $CD = DA$ in this case, because $\sqrt{(d-c)^2 + (a-d)^2} = \sqrt{(a-d)^2 + (b-a)^2}$ rearranges to $(d-c)^2 = (b-a)^2$ which is one of the equations we already had.
The answer is "if and only if $a - b + c - d = 0$".

(i) Bob's number is 1. Alice then realises her number is 2, because it can't be 0.

If Bob had any other number n, then Alice's number could be either n+1 or n-1, so Alice wouldn't know her number.

(ii) Write A for Alice's number, and B for Bob's number.

After Charlie's initial statement, the possibilities for (A,B) are

(2,1) (2,3)
(3,2) (3,4)
(5,4) (5,6)
(7,6) (7,8)

If Alice could see 1, 2, 3 or 8, she would deduce her number, so those possibilities are ruled out.

(3,4)
(5,4) (5,6)
(7,6)

If Bob could then see 3 or 7, he would deduce his number was 4 or 6, respectively, so those possibilities are ruled out.

(5,4) (5,6)

Hence Alice's number is 5.

(iii) Alice's statement implies Bob's number is not 1 or 10. Charlie's second statement implies that precisely one out of B-1 and B+1 is a square.

The possibilities are:

B=2 and A is 1 or 3
B=3 and A is 2 or 4
B=5 and A is 4 or 6
B=8 and A is 7 or 9

If Bob could see 1, 3, 2, 6, 7 or 9, he would know his number. Hence Alice's number is 4.

27

(i) Let's consider dates of the form d1 d2 / m1 m2 / 2 0 y3 y4.

Clearly m1 is 1 to avoid repetition of 0.

But then m2 is 0 or 1 or 2, each of which would be a repetition.

Hence there are no such dates.

(ii) As there are no such dates this century, let's try dates of the form d1 d2 / m1 m2 / 1 9 y3 y4.

The last possible year is 1987; then the last possible month is 06; and then the last possible day is 25.

This gives the date 25/06/1987.

(iii) As there are no such dates this century, let's try dates of the form d1 d2 / m1 m2 / 2 1 y3 y4.

Now m1 is 0 to avoid repetitions.

Then d1 is 3 to avoid repetitions.

But that leaves no possible value for d1.

Clearly there is no such date of the form d1 d2 / m1 m2 / 2 2 y3 y4.

For dates of the form d1 d2 / m1 m2 / 2 3 y3 y4, if m1 is 1 then m2 is 0 and there is no possibility left for d1. So m1 is 0 and d1 is 1.

Of the dates 1 d2 / 0 m2 / 2 3 y3 y4, the earliest possible year is 2345, then the earliest possible month is 06, and then the earliest possible day is 17.

This gives the date 17/06/2345.

(iv) Let's consider dates of the form d1 d2 / m1 m2 / 1 9 y3 y4.

Clearly m1 is 0.

If d1 is 3, then d2 is 0 or 1, either of which would be a repetition. Hence d1 is 2.

We therefore have dates of the form 2 d2 / 0 m2 / 1 9 y3 y4.

The remaining spaces can be filled with arbitrary distinct values from 3, 4, 5, 6, 7, 8, giving 6x5x4x3=360 possibilities. Each such possibility is a valid date.