

Examiners' Report:  
Final Honour School of Mathematics Part A  
Trinity Term 2023

October 26, 2023

## Part I

### A. STATISTICS

- **Numbers and percentages in each class.**

See Table 1.

Table 1: Numbers in each class

Range	Numbers					Percentages %				
	2023	(2022)	(2021)	(2020)	(2019)	2023	(2022)	(2021)	(2020)	(2019)
70–100	44	(59)	(53)	4(3)	(57)	30.77	(36.65)	(37.32)	(32.58)	(35.19)
60–69	67	(71)	(57)	(65)	(71)	46.85	(44.1)	(40.14)	(49.24)	(43.83)
50–59	25	(22)	(29)	(21)	(27)	17.48	(13.66)	(20.42)	(15.91)	(16.67)
40–49	4	(6)	(2)	(3)	(5)	2.8	(3.73)	(1.41)	(2.27)	(3.09)
30–39	1	(2)	(0)	(0)	(1)	0.7	(1.24)	(0)	(0)	(0.62)
0–29	2	(1)	(1)	(0)	(1)	1.4	(0.62)	(0.7)	(0)	(0.62)
Total	143	(161)	(142)	(132)	(162)	100	(100)	9(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**

Not applicable.

- **Marking of scripts.**

All scripts were single marked according to a pre-agreed marking scheme which was strictly adhered to. The raw marks for paper A2 are out of 100, and for the other papers out of 50. For details of the extensive checking process, see Part II, Section A.

- **Numbers taking each paper.**

All 143 candidates are required to offer the core papers A0, A1, A2 and ASO, and five of the optional papers A3-A11. Five candidates took six long options. Statistics for these papers are shown in Table 2 on page 2.

Table 2: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
A0	142	35.2	10.05	65.92	12.18
A1	143	33.41	10.05	66.4	14.8
A2	143	61.57	14.94	65.6	9.87
A3	67	29.73	9.86	65.31	14.04
A4	129	28.46	10.94	65.12	15.27
A5	83	29.66	9.6	64.59	14.15
A6	89	32.08	9.85	65.91	10.88
A7	61	28.34	10.17	64.1	15.31
A8	131	28.83	8.15	65.66	12.21
A9	70	34.09	10.38	66.67	14.08
A10	36	32.64	7.48	64.64	10.02
A11	49	25.45	12.01	62.82	12.67
ASO	142	32.46	9.01	67.09	13.2

### B. New examining methods and procedures

None.

### C. Changes in examining methods and procedures currently under discussion or contemplated for the future

None.

### D. Notice of examination conventions for candidates

The first notice to candidates was issued on the 21st February 2023 and the second notice on the 18th May 2023.

These can be found at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/ba-master-mathematics/examinations-assessments/examination-20>, and contain details of the examinations and assessments. The course handbook contains the link to the full examination conventions and all candidates are issued with this at induction in their first year. All notices and examination conventions are online at <https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions>.

## Part II

### A. General Comments on the Examination

#### Acknowledgements

- Haleigh Bellamy for her work in supporting the Part A examinations throughout the year, and for her help with various enquiries throughout the year.
- Waldemar Schlackow for running the database and the algorithms that generate the final marks, without which the process could not operate.
- Clare Sheppard and Charlotte Turner-Smith for their help and support, together with the Academic Administration Team, with marks entry, script checking, and much vital behind-the-scenes work.
- The assessors who set their questions promptly, provided clear model solutions, took care with checking and marking them, and met their deadlines, thus making the examiners' jobs that much easier.
- Several members of the Faculty who agreed to help the committee in the work of checking the papers set by the assessors.
- The internal examiners and assessors would like to thank the external examiners, Prof Neil Strickland and Prof John Billingham, for helpful feedback and much hard work throughout the year, and for the important work they did in Oxford in examining scripts and contributing to the decisions of the committee.

#### Timetable

The examinations began on Monday 12th June and ended on Friday 23rd June.

#### Mitigating Circumstances Notices to Examiners

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

### B. Strike Action

The marking boycott affected the marking of the papers A4 Integration and A5 Topology. Alternative markers were found for A4 Integration, and the completed raw marks for A5 Topology and the scripts were returned on the morning of the final board meeting. The A5 Topology examination scripts were checked and the marks imported after the final board, and the scaling for the marks agreed via confidential correspondence. The A5 marks were released alongside the rest of the Part A results.

#### Setting and checking of papers and marks processing

As is usual practice, questions for the core papers A0, A1 and A2, were set by the examiners and also marked by them with the assistance of assessors. The papers A3-A11, as well as

each individual question on ASO, were set and marked by the course lecturers/assessors. The setters produced model answers and marking schemes led by instructions from Teaching Committee in order to minimize the need for recalibration.

The internal examiners met in December to consider the questions for Michaelmas Term courses (A0, A1, A2 and A11). The course lecturers for the core papers were invited to comment on the notation used and more generally on the appropriateness of the questions. Corrections and modifications were agreed by the internal examiners and the revised questions were sent to the external examiners.

In a second meeting the internal examiners discussed the comments of the external examiners and made further adjustments before finalising the questions. The same cycle was repeated in Hilary term for the Hilary term long option courses and at the end of Hilary and beginning of Trinity term for the short option courses. *Papers A8 and A9 are prepared by the Department of Statistics and jointly considered in Trinity term.* Before questions were submitted to the Examination Schools, setters were required to sign off on a camera-ready copy of their questions.

The whole process of setting and checking the papers was managed digitally on SharePoint. Examiners adopted specific and detailed conventions to help with version checking and record keeping.

Examination scripts were collected by the markers from Exam Schools or delivered to the Mathematical Institute for collection by the markers and returned there after marking. A team of graduate checkers under the supervision of Haleigh Bellamy and Charlotte Turner-Smith sorted all the scripts for each paper, cross-checking against the mark scheme to spot any unmarked questions or part of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the marks scheme, noting any incorrect addition.

### **Determination of University Standardised Marks**

The examiners followed the standard procedure for converting raw marks to University Standardized Marks (USM). The raw marks are totals of marks on each question, the USMs are statements of the quality of marks on a standard scale. The Part A examination is not classified but notionally 70 corresponds to ‘first class’, 50 to ‘second class’ and 40 to ‘third class’. In order to map the raw marks to USMs in a way that respects the qualitative descriptors of each class the standard procedure has been to use a piecewise linear map. It starts from the assumption that the majority of scripts for a paper will fall in the USM range 57-72, which is just below the II(i)/II(ii) borderline and just above the I/II(i) borderline respectively. In this range the map is taken to have a constant gradient and is determined by the corners  $C_1$  and  $C_2$ , which encode the raw marks corresponding to a USM of 72 and 57 respectively. The guidance requires that the examiners should use the entire range of USMs. Our procedure interpolates the map linearly from  $C_1$  to  $(M, 100)$  where  $M$  is the maximum possible raw mark. In order to allow for judging the position of the II(i)/III borderline on each paper, which corresponds to a USM of 40, the map is interpolated linearly between  $C_3$  and  $C_2$  and then again between  $(0, 0)$  and  $C_3$ . Thus, the conversion of raw marks to USMs is fixed by the choice of the three corners  $C_1, C_2$  and  $C_3$ . While the default  $y$ -values for these corners were given above and are not on the class borderlines, the examiners may opt to change those

default values, e.g., to avoid distorting marks around class boundaries. The final choice of the scaling parameters is made by the examiners, guided by the advice from the Teaching Committee, considering the distribution of the raw marks and examining individuals on each paper around the borderlines.

The final resulting values of the parameters that the examiners chose are listed in Table 3.

Table 3: Parameter Values

Paper	C1	C2	C3
A0	(46.2,72)	(22.2,57)	(12.75,37)
A1	(42.6,72)	(23.1,57)	(13.27,37)
A2	(75.2,72)	(43.7,57)	(25.10,37)
A3	(38.6,72)	(19.1,57)	(10.97,37)
A4	(37.8,72)	(16.8,57)	(9.65,37)
A5	(37.2,72)	(20.7,57)	(11.89,37)
A6	(42,72)	(19.5,57)	(11.20,37)
A7	(37.4,72)	(19.4,57)	(11.14,37)
A8	(35,72)	(20,57)	(11.49,37)
A9	(44,72)	(21.5,57)	(12.35,37)
A10	(40,72)	(25,57)	(14.36,37)
A11	(39.2,72)	(19,60)	(4.42,37)
ASO	(39.2,72)	(22.7,57)	(13.04,37)

Table 4 gives the resulting final rank and percentage of candidates with this overall average USM (or greater).

Table 4: Rank and percentage of candidates with this overall average USM (or greater)

Av USM	Rank	Candidates with this USM or above	%
90.4	1	1	0.7
89.5	2	2	1.4
84.85	3	3	2.1
84.3	4	4	2.8
83.9	5	5	3.5
82.8	6	6	4.2
82.7	7	7	4.9
82.5	8	8	5.59
82.3	9	9	6.29
81.7	10	10	6.99
81.5	11	11	7.69
81.3	12	12	8.39
80.8	13	13	9.09
79.4	14	14	9.79
79.2	15	15	10.49
79.1	16	16	11.19
78.9	17	17	11.89
77.8	18	19	13.29
77.8	18	19	13.29
77.4	20	20	13.99
76.8	21	21	14.69
76.6	22	22	15.38
76	23	23	16.08
75.8	24	24	16.78
75.65	25	25	17.48
75.6	26	26	18.18
75.4	27	27	18.88
74.5	28	28	19.58
73.8	29	29	20.28
73.5	30	30	20.98
73.3	31	31	21.68
73	32	32	22.38
72.7	33	33	23.08
72.6	34	34	23.78
72.5	35	35	24.48
72.1	36	36	25.17
71.5	37	37	25.87
71	38	38	26.57
70.6	39	40	27.97
70.6	39	40	27.97
70.4	41	41	28.67
70.1	42	42	29.37

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
69.9	43	43	30.07
69.5	44	44	30.77
69.45	45	45	31.47
69.1	46	46	32.17
68.9	47	47	32.87
68.8	48	48	33.57
68.5	49	50	34.97
68.5	49	50	34.97
68.4	51	51	35.66
68.3	52	52	36.36
68.2	53	53	37.06
68.1	54	57	39.86
68.1	54	57	39.86
68.1	54	57	39.86
68.1	54	57	39.86
68	58	58	40.56
67.6	59	59	41.26
67.3	60	62	43.36
67.3	60	62	43.36
67.3	60	62	43.36
67.2	63	63	44.06
67.1	64	64	44.76
66.9	65	65	45.45
66.6	66	68	47.55
66.6	66	68	47.55
66.6	66	68	47.55
66.5	69	69	48.25
66.2	70	70	48.95
66.1	71	71	49.65
65.9	72	72	50.35
65.7	73	74	51.75
65.7	73	74	51.75
65.4	75	75	52.45
65.2	76	78	54.55
65.2	76	78	54.55
65.2	76	78	54.55
65.1	79	79	55.24
64.85	80	80	55.94
64.7	81	81	56.64
64.6	82	82	57.34
64.5	83	83	58.04
64.3	84	84	58.74

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
64.1	85	85	59.44
64	86	86	60.14
63.9	87	87	60.84
63.8	88	88	61.54
63.7	89	89	62.24
63.2	90	90	62.94
63.1	91	92	64.34
63.1	91	92	64.34
62.8	93	93	65.03
62.6	94	94	65.73
62	95	96	67.13
62	95	96	67.13
61.8	97	99	69.23
61.8	97	99	69.23
61.8	97	99	69.23
61.7	100	100	69.93
61.5	101	101	70.63
61.4	102	102	71.33
61.1	103	103	72.03
60.4	104	104	72.73
60.3	105	105	73.43
60.1	106	106	74.13
59.9	107	107	74.83
59.8	108	110	76.92
59.8	108	110	76.92
59.8	108	110	76.92
59.6	111	111	77.62
59.3	112	112	78.32
58.5	113	113	79.02
57.9	114	114	79.72
57.4	115	115	80.42
57.1	116	117	81.82
57.1	116	117	81.82
57	118	118	82.52
56.9	119	119	83.22
56.5	120	120	83.92
56.4	121	121	84.62
55	122	122	85.31
54.4	123	125	87.41
54.4	123	125	87.41
54.4	123	125	87.41
54.2	126	126	88.11

Table 4: Rank and percentage of candidates with this overall average USM (or greater) [continued]

Av USM	Rank	Candidates with this USM or above	%
54.11	127	127	88.81
53.8	128	129	90.21
53.8	128	129	90.21
53.5	130	130	90.91
53.3	131	131	91.61
51.9	132	132	92.31
51.2	133	133	93.01
50.6	134	134	93.71
50.4	135	135	94.41
50.3	136	136	95.1
46.5	137	137	95.8
45.4	138	138	96.5
45.2	139	139	97.2
40.4	140	140	97.9
34	141	141	98.6
29.1	142	142	99.3
22.57	143	143	100

### Recommendations for Next Year's Examiners and Teaching Committee

The process seemed to work well this year. To try to optimise things even further next year, it may be helpful for the next Chair of Examiners to remind the setters of various papers to use a consistent system of names for folders containing draft versions of exam papers, e.g. Smith220126. This will help to minimise the risk of various changes to the papers being overwritten in future edits.

## B. Equality and Diversity issues and breakdown of the results by gender

Table 5, page 10 shows percentages of male and female candidates for each class of the degree.

Table 5: Breakdown of results by gender

Class	Number								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	4	40	44	7	52	59	5	48	53
60–69	19	48	67	23	48	71	21	36	57
50–59	11	14	25	8	14	22	15	14	29
40–49	3	1	4	4	2	6	1	1	2
30–39	0	1	1	0	2	2	0	0	0
0–29	2	0	2	1	0	1	0	1	1
Total	43	118	161	42	100	142			

  

Class	Percentage								
	2023			2022			2021		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
70–100	10.26	38.46	30.77	11.9	48	37.32	24.39	36.26	30.32
60–69	48.72	46.15	46.85	50	36	40.14	53.66	47.25	50.45
50–59	28.21	13.46	17.48	35.71	14	20.42	17.07	15.38	16.22
40–49	7.69	0.96	2.8	2.38	1	1.41	4.88	1.1	2.99
30–39	0	0.96	0.7	0	0	0	0	0	0
0–29	5.13	0	1.4	0	1	0.7	0	0	0
Total	100	100	100	100	100	100	100	100	100

## C. Detailed numbers on candidates' performance in each part of the exam

Individual question statistics for Mathematics candidates are shown in the tables below.

### Paper A0: Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.24	18.43	6.14	123	2
Q2	16.06	16.24	6.62	46	1
Q3	17.33	17.4	4.54	114	2

**Paper A1: Differential Equations 1**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.4	13.49	5.87	115	2
Q2	18.4	18.4	5.29	142	0
Q3	21.24	21.93	5.99	28	1

**Paper A2: Metric Spaces and Complex Analysis**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.42	16.5	3.77	103	3
Q2	15.56	15.56	5.34	109	0
Q3	9.82	10.45	4.75	67	7
Q4	14.43	14.63	5.75	109	2
Q5	17.8	17.96	4.64	116	1
Q6	15.32	15.4	5.95	67	1

**Paper A3: Rings and Modules**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.44	14.52	5.09	63	1
Q2	14.69	14.98	5.58	44	1
Q3	15.07	15.48	6.96	27	1

**Paper A4: Integration**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.55	11.82	6.26	90	3
Q2	14.7	14.81	5.76	124	1
Q3	17.11	17.5	6.34	44	1

**Paper A5: Topology**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.31	15.31	4.83	81	0
Q2	13.94	14.02	5.82	64	1
Q3	15.48	15.48	5.09	21	0

**Paper A6: Differential Equations 2**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.86	18.86	4.88	88	0
Q2	12.56	12.71	6.28	49	1
Q3	12.8	14.3	6.21	40	6

**Paper A7: Numerical Analysis**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.93	12.93	6.1	55	0
Q2	15.68	16.09	5.97	58	2
Q3	9.4	9.44	6.64	9	1

**Paper A8: Probability**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.13	14.64	5.54	99	5
Q2	14.12	14.6	4.78	94	5
Q3	13.94	14.06	3.36	68	1

**Paper A9: Statistics**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.04	15.21	5.55	48	1
Q2	14.31	14.51	5.62	35	1
Q3	19.81	20.14	5.56	57	1

**Paper A10: Fluids and Waves**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.37	17.37	3.93	19	0
Q2	15.11	15.82	4.95	34	2
Q3	16.16	16.16	4.87	19	0

**Paper A11: Quantum Theory**

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	12.69	12.87	6.87	47	1
Q2	10.13	10.38	5.27	37	1
Q3	18.43	18.43	5.11	14	0

## Paper ASO: Short Options

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.93	13.93	5.37	44	0
Q2	14.82	15.03	4.93	32	1
Q3	18.86	18.86	5.42	14	0
Q4	12.75	12.75	4.19	4	0
Q5	17.97	18.38	6.27	84	2
Q6	13.91	13.91	4.46	43	1
Q7	17.6	17.6	5.11	30	0
Q8	19.6	23.75	9.34	4	1
Q9	14.87	15	2.94	29	1

## D. Comments on papers and on individual questions

The following comments were submitted by the assessors.

### Core Papers

#### A0: Algebra 1

**Question 1** Surprisingly, in part (a), (i) many students did not state the PDT correctly. Some students implicitly assumed that the field is algebraically closed and stated the theorem under this assumption, and other omitted important conclusions. Both of these resulted in partial or full reduction of mark. In part (ii), the most common problem was that students wrote the final answer implicitly without findings explicitly the requested kernels. Other mistakes included using the imaginary unit  $i$  (or other meaningless symbols) which is meaningless in the field of 3 elements. Calculation errors were treated with more tolerance.

In part (b), (i) almost all students defined correctly the requested notions, all though some confused  $\mathbb{C}$  with  $\mathbb{R}$ . In part (ii), most students found correctly the formula, but many students did not explain why the vectors which span  $U_0$  are linearly independent. Most students wrote a correct proof in part (iii), and the few who did not either left a blank space or wrote an argument which has nothing to do with the question. In part (iv) some students left a blank space, but most did well. Some did not explain clearly enough how to deduce the lower triangular statement from the upper one (or vice versa).

**Question 2** This was the least popular question, done by a fairly small proportion of students. Part (a)(ii) was very mildly disguised bookwork. The group of students taking the question seemed to fall into two parts: those who had learned the bookwork (and did quite well), and those that hadn't but perhaps didn't like the look of one of the other two questions (and did not do very well). I was pleased though that those students who had studied carefully this part of the course were able to score very highly on the question. I was amazed though by the number of students who, in a question about nilpotent maps, spent pages trying to calculate the characteristic polynomial of a  $4 \times 4$  matrix which you are told is nilpotent. (It's  $x^4$ .)

**Question 3** This was a very popular question done by most students. And it was done very well overall. Parts that caused the most problems were (b)(ii) and (iv), and (c)(i)

(where alternative longer solutions not using (b) were accepted too). But many students gave complete or near complete solutions. I was happy about this as I didn't think a question on adjoints would be so popular and done so well. (I suppose for many it was a choice between adjoints and nilpotent maps, and the adjoints won out.)

## A1: Differential Equations 1

**Question 1** Qn1 has been attacked by majority of students. Parts (a) and (b) have been done well in many scripts. The most difficult part for students was the last one, part (c), although technically it was easy. The main point of (c) is to get an estimate of the maximum of modulus of solutions that are not a result of successive approximations.

**Question 2** Part (a) Some of the candidates do not understand the definition of stable at one point. They did not clarify clearly. Most of the candidates write clear and find critical points. This is not hard for them. Part (b) is harder. Most of the candidates can write well for the first two questions. They understand the definition of characteristic projections and characteristic curves, but some of them do not understand how to solve the ODE, which is the so-called characteristic equations. Therefore, they did not do well for (iii). Only a few students can find domain of the existence and uniqueness of the explicit solutions, and find points where  $f$  blows up.

**Question 3** Part (a). Candidates generally knew how to prove by contradiction for proving Maximum principle. And many candidates construct many possible examples, which is very interesting. The proof of (iii) is standard. Most of the candidates can understand clearly what they are proving and it is clear. Part (b). The question is not that difficult. Candidates know how to do integration by parts and do estimates. For (ii), it is a standard energy approach. Most of the candidates can prove  $u = 0$  by using Gronwall's inequality.

Overall, this is a successful paper with lots of concepts covered and can distinguish whether the Candidates understand the concepts or not.

## A2: Metric Spaces and Complex Analysis

**Question 1** Overall, the question was done successfully by most candidates. Many candidates obtained 20 marks or more, and several obtained 25.

Part a) Here, i) and ii) were largely correct. In iii), many candidates only proved one direction of the implication, receiving 3 out of 6 marks. In some cases it was clear that the candidate ran out of time, whereas in others it was apparent that the candidate simply forgot to prove the other direction.

Part b) In i) many people obtain 4 out 6 marks, by proving everything asked for them except the triangle inequality. The earlier parts were fine (though some candidates forget to prove that  $\rho(F_1, F_2) = 0 \implies F_1 = F_2$ ), but many answers either do not attempt the triangle inequality, or make general appeals (e.g. "the maximum preserves the triangle inequality"), or write inequalities which are incorrect. Few candidates actually expand the sup/inf definition and operate rigorously. Curiously, most candidates start with  $\rho(F_1, F_2) + \rho(F_2, F_3)$  and attempt to bound it below by  $\rho(F_1, F_3)$ , rather than the other way around, which I find more intuitive. In ii), most are able to prove that  $G$  is closed and non-empty, but not many succeed in proving  $\rho(F_n, G) \rightarrow 0$ , receiving only 4 marks out of 6. A number

of candidates attempt to prove that  $G$  is a singleton, not realising that the diameter of  $F_n$  needs not tend to zero. Similarly, in iii), a number of candidates again construct arguments assuming  $G$  is a singleton.

**Question 2** Overall, 2(a) was well done; almost all students had the correct definitions of compactness and a majority of students were able to at least obtain partial marks for correctly setting up 2(a)(ii). There were two common issues in 2(a)(ii). First, some students did not prove the property that the intersection of nested closed sets in a compact set is nonempty. Second, there were some issues with the order of logical quantifiers. In particular, given an arbitrary sequence  $(x_n)_{n=1}^{\infty}$ , some students *first* fixed some  $\epsilon > 0$ , and *then* found a subsequence such that the tail was bounded by this fixed  $\epsilon$ . However, this does not imply that the tail will be bounded by some smaller  $\epsilon$ . This was only an issue when students attempted the proof by building a subsequence using finite covers of  $\epsilon$ -balls, and didn't really affect the other proof methods.

Question 2(b)(i) was also very well done. The same issue regarding the order of quantifiers from above was more common in 2(b)(ii). By first fixing an  $\epsilon$ , some students tried to find subsequences which were contained in some  $\epsilon$ -ball for finitely many coordinates (since the tail will already be smaller than  $\epsilon$ ), while others who iteratively constructed subsequences which converged in each coordinate only did this for finitely many coordinates; both approaches (incorrectly) avoided the diagonalization argument. Some students also did not prove the sequence they constructed converges, even if their construction was correct.

Question 2(c) was quite tricky, and students had issues with all the subparts. Some students immediately claimed (without proof) that since a subsequence of  $\phi^{(n)}(x)$  converges, it would converge to  $x$ . With surjectivity, students often made claims without enough justification.

### Question 3

Overall, the question was not done very well by most of the candidates. Few candidates obtained 20 or similar, while the majority obtained lower marks.

Part a) This part was mostly correct.

Part b) Here students had several difficulties; only few of them took the right path, though they did not complete it.

Part c) In this part the majority of the students understood the main point, but not everyone completed it.

Part d) Many candidates confused the proposition used to prove the Identity Theorem with the actual result.

Part e) This part was mostly correct.

**Question 4.** Q4 was a popular question. Part (a) was harder than it looked, and many candidates lost marks for not being precise enough with their definitions. For example, for subpart (i) it was not enough to state the  $f$  is holomorphic in a neighbourhood of  $a$ : it is also necessary to say that  $f$  is not holomorphic at  $a$  itself. Part (b), pure bookwork in the form of the Casorati-Weierstrass theorem, was done well by those candidates who studied the notes thoroughly, and it was challenging for all others. Part (c) was straightforward but computing  $f'/f$  when  $f$  is written as a Taylor series is a mess: it is much easier to write  $f = (z - a)^{-n}g$  for some holomorphic  $g$  and calculate the Laurent expansion of  $f'/f$ . Part (d) was tricky: the majority of candidates got as far as applying part (c) to the given function  $f = \exp(g)$ , but then could not justify why it is impossible for  $g'$  to have a simple pole at

the singular point. The best solution, found by only a handful of students, was to integrate  $g'$  along small circular path around the singularity and apply the Fundamental Theorem of Calculus to derive a contradiction.

**Question 5.** Part (a) was generally done well. Some of the candidates did not clarify the removable singularity at 0. Part (b) some of the candidates did not know how to calculate this integral and did not know how to use Cauchy's Residue Theorem. Part (c), some of the candidates did not point out standard semicircular contour and did not know how to use Cauchy's Residue Theorem. There is a common mistake that almost all the candidates fail to take half of the number at the end, taking real parts and noting that the function is an even function. Overall, this is a successful paper with lots of concepts covered and can distinguish whether the candidates understand the concepts or not.

**Question 6.** Overall, the question was done okay by most candidates.

Part a) Here, i) and ii) were often correct when done at all.

Part b) i) Was done successfully by many people. ii) Was sometimes not attempted and sometimes people obtained 4/6 points as they did not calculate  $f^{-1}$ .

Part c) i) Was solved successfully by most people that attempted it. ii) Was rarely solved. Sometimes people just wrote yes or no without explanation.

## Long Options

### A3: Rings and Modules

**Question 1** Q1 was a long question with a good spread of marks. The aim was to show that there are Abelian groups (*e.g.*  $C_5$ ) that do not occur as the group of units of a ring. For the final part of this the question assumed that  $X^p - 1$  is square-free (for  $p > 2$ ). This can be proved by noting that  $X^p - 1$  and its formal derivative  $pX^{p-1}$  are coprime, but this was not covered in the course and was omitted to avoid an already long question being over-long.

For (a)(ii) a number of answers showed that the projection of  $I$  onto the  $i$ th factor was an ideal but failed to justify that  $I = I_1 \times \cdots \times I_n$  – compare this with the fact that the unit disc is not equal to the product of its projections onto the  $x$  and  $y$  axes. (a)(iii) was harder, with the point being that if  $|U(R)|$  is odd then since  $-1$  is always a unit we must have  $-1 = (-1)^{|U(R)|} = 1$  by Lagrange's Theorem and hence  $R$  has characteristic 2. (b) was mostly well done, though many solutions to (b)(i) were unnecessarily long and in (b)(ii) some misread the ideal  $\langle x_1 \cdots x_n \rangle$  as being  $\langle x_1, \dots, x_n \rangle$ . Part (c) served well to identify those who had firmly grasped the rings part of the course.

**Question 2** Q2 was a question intended to give an example of a module – the Baer-Specker group – which might seem to be free but which in fact is not. The more routine material of part (a) was well done and could be done in fairly short order; it set up the tools necessary for (b). (b)(iv) was hard with very few considering  $0 \neq \pi(w) = \pi(w - f_i)$  and using  $\mathbb{Z}$ -linearity to derive the contradiction.

**Question 3** Q3 was perhaps the most mixed of the questions – certainly the answers had the highest variance in score. The original intention with part (b) was to show that if  $a, b, c \in \mathbb{Z}$  have  $bc = a^2 + 1$  then there is  $F \in \text{GL}_2(\mathbb{Z})$  such that

$$F^{-1} \begin{pmatrix} a & b \\ -c & -a \end{pmatrix} F = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The matrix being conjugated on the left has minimal polynomial  $X^2 + 1$  so part (b) of the question applies, and the matrix  $F$  can be constructed from the matrices  $U$  and  $V$ . This would have been too hard as a question, and so after useful feedback the given Q3 emerged; it proved very suitable.

For (a)(ii) some failed to realise that  $A \in M_n(\mathbb{Q})$  for general  $n$ , though the explicit statement that  $n = 2$  in (a)(iii) was intended as an extra hint at the generality of (a)(ii). For (b)(i) many remembered the proof that the Gaussian integers are a Euclidean Domain from the problem sheets and for (b)(iii) there were a number of different approaches which all showed a keen understanding of the material. The most common error was to take  $Q = iI + A$ , however this is not invertible. (b)(iv) was quite straight-forward.

#### A4: Integration

**Question 1.** Question 1. In part (a), almost all students defined correctly the notion of a sigma algebra, with exception of a few students which replaced invariance under countable unions by finite unions. The majority of students also correctly defined what is a Lebesgue measurable function, although there were a few students who demonstrated confusion between Borel and Lebesgue sigma algebra. Part (ii) was also quite satisfactory. A few students made implicitly the additional and unnecessary assumption that the functions are invertible and/or continuous.

In part (b), (i) the most common mistake was that students did not distinguish between the case where  $m(E) < \infty$  and  $m(E) = \infty$ . In part (ii) the most common mistake was that students did not prove equivalence between  $m(E) < \infty$  and the condition stated in the question (very few students proved both directions). Other more serious mistakes included false statements such that any compact set can be written as a union of an interval and a null set.

In part (c), (i) most students gave a correct answer and a correct counterexample. Some students demonstrated a confusion between the Cantor and Vitali sets, and made the false claim that the Cantor set is not Lebesgue measurable. Much less students did well on part (ii). Some students just left a blank space or made a false claim of a counterexample (for example by taking product of 2 "exotic" null sets and mistakenly thinking they are not measurable). Among the students that demonstrated a correct approach, a few did not indicate clearly enough where exactly Tonelli theorem is being used.

In part (d), most students realized that one should use the fact that the measure of  $E$  can be written as the infimum over all countable covers by intervals, but the application of this was at times incorrect or with missing details. Many students did not explain clearly enough how exactly to choose the requested interval. A more serious mistake included a false claim (usually also with no attempted proof) that any set of positive measure must contain an interval.

**Question 2.** a(i) was generally very well answered, though several candidates made things more difficult than needed by considering substitutions  $y = \frac{1}{x}$  or  $y = \frac{1}{x^3}$ . This doesn't actually make the problem easier (though typically these substitutions were well done), and it's faster to note that  $\sin(1/x^3)$  is bounded on  $(0, 1)$  and  $|\sin(1/x^3)| \leq 1/x^3$  on  $[1, \infty)$ . a(ii) was also generally well answered. However a number of candidates claimed that  $1/x$  is an antiderivative of  $\log x$  and obtained the incorrect result that  $\log x$  is not integrable on  $(0, 1)$ .

The strategy of performing a polar substitution in (b)(i) was identified and implemented by most candidates, but not all took care to justify the various steps involved by clearly indicating the polar substitution theorem and how the theorems of Tonelli and Fubini are used to support the calculations. In b(ii) many candidates correctly used the continuous parameter DCT to obtain continuity of  $F(t)$  on  $[0, \infty)$  and hence at 0. Establishing differentiability proved much more problematic, with a number of candidates either failing to use a local version of the differentiation theorem (often claiming incorrectly that expressions of the form  $\frac{K}{x^2}e^{-x^2}$  would be integrable on  $(0, \infty)$ ), or struggling to get the estimates in a local version to work out. The very last part of the question contained an error: it should have asked for  $F'(t) = -2F(t)$ . The mark scheme was adjusted so that it was not necessary to compute  $F'(t)$  in order to get full marks, but sensible attempts were rewarded. I'm sorry to students taking this paper for this error, which will be corrected for students revising in future years.

**Question 3.** Part (a) was generally done well. Most candidates can prove Minkowski's inequality. Many of them use the convexity of function and separate the discuss when the function is zero nor not. Some of the candidates did not distinguish the normed spaces for  $p = \infty$  or  $p < \infty$ . For the completeness of Lebesgue's space, some of the candidates did not suppose the sequence is Cauchy and then prove its convergence, which is not correct. Part (b) was not that easy. Some of the Candidates write something meaningless. And they make the wrong statements and construct wrong counter example. For the second statement, some of the candidates did not use Cauchy Swartz' inequality and DCT which is not correct. Part (c) , a few number of candidates write nothing, but some of the candidates write well and are very clear what they are going to prove. Overall, this is a successful paper with lots of concepts covered and can distinguish whether the Candidates understand the concepts or not.

## A5: Topology

**Question 1** Virtually all candidates attempted this question.

1a i. Many candidates gave an incorrect definition for connected subsets.

1a ii, 1a iii generally well done.

1a iv Several different examples were given, generally correct.

1a v. Candidates found this question challenging. Several candidates tried to prove this using only that the space is Hausdorff-which is of course false.

1b i. Many candidates realized that they could use 1a iii and answered this correctly.

1b ii Generally well done.

1b iii Candidates found this challenging but several managed to give a valid argument.

**Question 2** Many candidates attempted this question.

2a i. Generally well done.

2a ii The proof that  $f$  is continuous was often wrong or missing, and similarly for  $f^{-1}$ .

2a iii. Students found hard to give a formal proof of this. Some candidates got partial credit for describing a correct map without a formula.

2a iv. Mostly well done.

2bi, bii Done by most candidates.

2biii Sometimes students gave a map that was not actually a homeomorphism and got no marks.

2biv. Some students gave counterexamples with  $X$  not a metric space as required.

**Question 3** 28 candidates attempted this question.

3ai. Well done but some students did not define the standard  $n$ -simplex or were not very precise in their definition of the quotient space.

3aaii. Some arguments were not formal enough, for example they did not define precisely the two subcomplexes and got only partial credit.

3aiii, aiv Well done

3bi, bii Generally triangulations were correct but the proof of the second part was not always clear.

3biii Students found this hard and some gave incomplete proofs, but there were some fully justified solutions too.

## A6: Differential Equations 2

**Question 1** - was the most popular question and was attempted by almost every candidate. This question was generally done well, especially parts (a) and (b). Quite a few candidates made slips with the algebra to find the Green's function - a very common minor error being to inadvertently flip the sign of the jump condition on the function's gradient. A common issue in part (c) was not to note the boundary conditions and assume a simpler homogenous orthogonality condition.

**Question 2** - the attempts at this question were wide ranging. Many candidates got lost in the algebra to deal with the Frobenius series in part (a), due in part to the series not being centered at  $x = 0$  (a simple shift of variables, as shown in lectures, would have helped many). Part (b) was done better than part (a), although the connection to the eigenfunctions  $P_n$ , and particularly to  $P_0$ , seemed to be lost on many (with more general statements given about orthogonality to the homogenous adjoint solutions). Many candidates did not give much answer for part (c), but those that did have a significant attempt generally did quite well. Some candidates left their answer in terms of inner products involving the  $P_n$  that were not noted to be zero by the orthogonality condition in (b).

**Question 3** - this question proved surprisingly challenging. Part (a)(i) was mostly fine, though a common issue was to over-use Taylor expansions of  $\sin(x)$  even for solutions that are not close to  $x = 0$  (though most candidates were still able to obtain the correct answers with appeals to periodicity). Very few candidates realised that a general scaling (leading to  $x \sim \varepsilon^{1/3}$ ) was needed to locate the root near  $x = 0$  in part (ii). In part (b), many candidates had the boundary layers in the wrong locations, and there was quite a lot of confusion about which boundary conditions to apply where (for example some candidates made the outer solution in part (i) satisfy the boundary condition at  $x = 0$  as well as having a boundary layer there). This went together with frequent confusion about how to match between inner and outer solutions. Nevertheless, this part was completed fully by a number of candidates and generally seemed to be found easier than part (a).

## A7: Numerical Analysis

This appear to have been a reasonably fair paper with marks across a wide range. Most candidates seem to have been reasonably prepared, though there were three particularly poor scores where candidates seem not to have engaged with the material of the course.

**Question 1** Q1 on eigenvalue computation was attempted by the majority of candidates. The standard bookwork in part (a) most often attracted full marks, though some candidates failed to produce a correct proof for Gershgorin's Circle Theorem. Part(b) was also bookwork that was done reasonably by many though there were several misconceptions. Part(c) on inverse iteration asked for synthesis which many candidates were able to provide at least in part. The unseen example in part(d) proved a challenge for many though there were some excellent attempts including a few with full marks. It was somewhat disappointing that the majority of demonstrations of the invertibility of the given matrix reverted to a direct computation of the determinant using cofactors when a simple application of the Gershgorin Theorem of part (a) immediately shows this.

**Question 2** Q2 on interpolation was attempted by all candidates bar one and there were several high scores. The standard bookwork in part (a) most often attracted full marks, though some candidates were careless about polynomial degree and some produced only some of what was asked for. Only about half of candidates successfully completed part(b) with rather more than one might expect not realising that  $x^k$  interpolates itself at any data points and thus if of low enough degree to be a candidate for the Lagrange Interpolating polynomial must be it because of uniqueness (as proved in part(a)). Even if this was realised, fewer observed that the requested result followed from simple equating of a polynomial coefficient with rather too many attempting brute force calculational proofs none of which was successful. The unseen part (c) on rational interpolation was actually quite well done by those who attempted it.

**Question 3** Q3 on numerical methods for Ordinary Differential Equations attracted only a handful of serious attempts even though it was largely the application of standard ideas to a particular numerical method.

## A8: Probability

*See Mathematics and Statistics report.*

## A9: Statistics

*See Mathematics and Statistics report.*

## A10: Fluids and Waves

**Question 1.** This question, and particularly the first two parts, was generally well done. In part (c) many students forgot to apply the pressure boundary condition at  $r = R(t)$  and very few were able to spot the first integral suggested by the hint. Finally, students did not always use both initial conditions to determine the constant of integration and hence determined an incorrect integral for the collapse time.

**Question 2.** This question was attempted by all candidates. Part (a) was well done, but in part (b) relatively few candidates were able to show that the two stagnation points lie on

the required circle. (Candidates tended to write down the general solution of a quadratic, which is tricky to deal with; a better strategy was to spot that the condition for a stagnation point simplifies and can be ‘square-rooted’ to give two linear equations, one for each of the stagnation points.) Part (c) was generally well done, albeit with errors in the application of the residue theorem. In part (d), very few students appreciated that the calculation of the complex potential from part (a) needed to be modified because now  $b(t) \in \mathbb{C}$ . (This simplifies the calculation significantly because then the denominator is a function of  $a^2 - |b|^2$ .)

**Question 3.** Part (b) of this question was generally done very well. However, many candidates struggled to provide the definitions in part (a) — particularly the meaning of *dispersive* versus *non-dispersive* — which then held them back in the latter stages of part (c).

### A11: Quantum Theory

All candidates but one chose Question 1. A little more than 1/4 of the candidates chose Question 3, which was conceptually the most advanced and unfamiliar, but technically not very demanding. The average score of this group of candidates was high and the separate averages on Question 1 and Question 3 only differed by about half a mark. A little less than 3/4 of the candidates chose Question 2, and their score was considerably lower on average and their separate averages on Question 1 and Question 2 were within one mark.

**Question 1** In part a) a sizeable fraction of candidates did not find the correct quantisation of  $k$ , which then often derailed their attempts in the later parts of the problem. If a candidate found the correct answer for b) (i), they almost always worked their way through the rest of part b) with ease, indicating that they had a good grasp of continuous time evolution and measurement in quantum theory. Only a few candidates engaged with part c) substantially, but those who did earned a good fraction of the marks.

**Question 2** The marks on this question were low; candidates struggled with part a), which was intended as bookwork. In particular many did not reproduce the relations  $\psi_{n_1, n_2}(x_1, x_2) = \psi_{n_1}(x_1)\psi_{n_2}(x_2)$  and  $E_{n_1, n_2} = E_{n_1} + E_{n_2}$  even though this was covered in lectures and similar ideas have featured several times throughout the course. We also note that this question was very similar to Question 2 on the 2021 exam. Some candidates got lost in computing commutators without a strategy in part d). Only very few candidates got to the challenging part e) question.

**Question 3** The marks on this question were very high, but it seems that only the most prepared candidates chose this question. The conceptual difficulty and unfamiliarity of this problem was balanced with its relative technical simplicity. Part a) of this question was bookwork. About half of the candidates used nonstandard conventions for Pauli matrices; they still got full marks. Part b) was seen before, but not practiced extensively. The subquestions led the candidates through the derivations in small steps, which they followed with ease. The only point of failure was not imposing the condition  $J_- = (J_+)^*$ . Part c) was conceptually the least familiar question in the exam, with the Hamiltonian being a finite dimensional matrix and the continuous time evolution interrupted by multiple measurements, but candidates navigated the conceptual part very well. Some lost time on finding the eigenstates of  $J_1$  with eigenvalue  $\hbar$ .

## Short Options

### ASO: Q1. Number Theory

Many candidates found (a)(i) challenging. The fact that the group of units is a cyclic group of order  $p-1$  was mentioned, but many candidates did not use this to determine the number of solutions. Another approach was to use FLT to reduce to  $k < p$ , but many candidates still had problems determining the number of solutions after that. In (a)(ii) most candidates did not observe that given any solution one can multiply by a solution to (i) to get another solution thus giving the same number of solutions as in (i) if solutions exist. (b) was done well by most candidates. Very few candidates managed to prove (c)(i). Most candidates who attempted this part of the question realised that any solution mod  $p^l$  is a solution mod  $p$ , but very few tried to use induction to show the other direction by lifting solutions from mod  $p^k$  to mod  $p^{k+1}$ . Many candidates solved (c)(ii) even if they did not solve c(i).

### ASO: Q2. Group Theory

This question was about defining groups in terms of generators and relations. In particular it concerned showing that  $SL(2, Z)$  could be generated by two elements.

Most people got the definition in (a) of group presentations in terms of generators and relations roughly correct, though many were too sketchy on details.

Part (b) on applying the Euclidean algorithm was fairly well done, but again many candidates didn't give enough detail in part (c). There was some confusion about the group whose presentation had generators  $u, v$  with relations  $u^2 = v^3 = 1$ . Some people thought this was the direct product  $C_2 \times C_3$  while of course it is the much larger free product as we don't know  $u$  and  $v$  commute. Part (d) was generally found difficult though a few candidates made good progress.

### ASO: Q3. Projective Geometry

This question was about the geometry of conics.

Candidates generally performed well on this question. The bookwork on classification of conics and intersection of projective lines was mostly well understood.

Most people got the idea in part (b) that the set of quadratic homogeneous polynomials in  $x, y$  with repeated factors itself forms a conic in the projective space of all such polynomials, defined by vanishing of the discriminant. Some forgot to show this conic was nonsingular.

Part (c) created problems in some cases but most people realised that tangency was equivalent to the line meeting the conic in a unique point and used this to solve the question.

In part (d) most people got the rough idea of the factorisation but many failed to account for special cases. The last part was generally done well.

### ASO: Q4. Multidimensional Analysis and Geometry

Relatively few candidates attempted this question, but for the most part those that did made a good attempt at it. As such, although no candidate scored perfectly on the question, every part of the question was solved by some candidate. Part (a) was generally well-answered, and

candidates lost marks largely for not providing sufficient justification for the claims made in their answers. Part (b) was also largely well-answered, with the most challenging point being the inclusion  $T_eO(E) \subseteq \mathcal{A}(E)$ . Part (c) was the most demanding part of the question, with most candidates failing to appreciate that  $\exp: \mathcal{L} \rightarrow \mathcal{L}$  is not injective.

#### **ASO: Q5. Integral Transforms**

a) Part (a) was generally well done, with most candidates correctly recalling the definition of the distributional derivative and using this to identify the derivative of the Heaviside distribution. The derivatives of  $|x|$  caused more problems, with some candidates considering  $x > 0$  and  $x < 0$  separately without commenting on the ambiguity at  $x = 0$ . Those who saw that  $|x|$  could be written in terms of the Heaviside function were generally able to use the earlier parts of the question to find the solutions with little difficulty.

b) i) Most candidates found the Laplace transform of the function, but the discussion around the values of  $p$  for which the transform existed was occasionally lacking. ii) Candidates who recalled the Laplace Convolution Theorem generally had few difficulties with this part, while a few were able to derive the solution through alternate means. Those who found the Laplace transform of  $I(x)$  generally had few issues recognising its relation to the solution of the required equation, and were able to solve the problem by inverting it and setting  $x = 1$ .

c) i) Generally well done - the most common problem here was in calculating the Fourier transform of the original equation. Those who completed this usually recognised why its relationship to the original equation required  $\hat{f}(s) = \lambda f(s)$ . ii) Most candidates did well in this part, with the most common cause of lost marks being to assume the initial statement that  $\hat{f}(0) = 1$  rather than showing it as asked. The subsequent parts of the question were well done. iii) This caused surprisingly many problems, with several candidates unable to correctly find the Fourier transform of  $g(x)$ . Those who found it correctly generally went on to find the correct solutions to the question.

#### **ASO: Q6. Calculus of Variations**

Overall the question seemed to work reasonably well. There was a decent spread of marks, and it seemed to successfully distinguish stronger candidates from weaker ones. Pleasingly, the vast majority of candidates produced a good answer for part (a) of the question, which demonstrated that they'd understood the basic ideas in the course. Parts (b) and (c) were both more challenging, and distinguished between stronger candidates. Slightly more candidates than expected failed to make much progress on either of these later part, so there were a few too many candidates who produced a good solution to (a) and little for either (b) or (c), indicating that part of them should perhaps have been pitched a little bit lower.

#### **ASO: Q7. Graph Theory**

This question was, in the assessor's view, a successful one. Part (a) tested a central topic in the course (Dijkstra's algorithm) in a way that was accessible to almost all students. Part (b) started off gently, with a request for reasonably straightforward proof, but ended with a more challenging final part.

The assessor was impressed by the quality of the answers, with most students providing clear and accurate solutions to most of the question.

Part (a) was very well done, with the majority being able to recall Dijkstra's algorithm and to implement it in the example. Parts (b)(i) and (b)(ii) were also generally well done. Almost all students had the right idea, but some struggled to formulate their arguments precisely. Part (b)(iii) provided the most challenge, and the majority of students gained less than half marks on this question. Most successful students followed the expected solution (where the algorithm is modified so that an edge is removed only if the resulting subgraph is connected). However, there were other correct solutions which started from a union of shortest maximum-length paths, and then removed further edges suitably until the subgraph formed a tree. A few students attempted to modify Dijkstra's algorithm to solve (b)(iii), which is possible, but generally the answers here were incomplete.

There was one minor inaccuracy in the question. In (b)(i), the candidates were required to prove a statement about the graph  $G_n$ . It was the setter's intention that this graph  $G_n$  refers to the output from the algorithm. However, the output is in fact  $G_{n+1}$ . Almost all students interpreted the question in the way that was intended. In fact, the question is correct as stated because necessarily  $G_n = G_{n+1}$  (and this can be easily proved). However candidates were certainly not required to make this observation. The assessor is confident that this minor inaccuracy caused students no difficulties.

### **ASO: Q8. Special Relativity**

The course is for distinguishing relativistic spacetime from Galilean spacetime. Most of the students who take the exam understand the material very well. For the book work part, their performance is nearly perfect. They also did fine in the application part, which is not technical this year. There are still several students who did poorly even in the bookwork part.

### **ASO: Q9. Modelling in Mathematical Biology**

(a) The biological explanation was quite well done but some candidates did not stress that the  $r$ 's are linear growth rates (strictly speaking, per capita linear growth rates) and a number of candidates proposed this as a predation model rather than a competition model. The rest of this question was very well done.

(b) Everyone knew how to do linear stability analysis but a surprising number of candidates did not seem to realise that the eigenvalues of a  $2 \times 2$  matrix, when one off-diagonal term is zero, are simply the diagonal terms. Much time was lost doing unnecessary time-consuming calculations. Virtually all students struggled to draw phase planes.

(c) Few students attempted this part.

## **E. Names of members of the Board of Examiners**

- **Examiners:**

- Prof. Konstantin Ardakov (Chair)
- Prof. Alan Lauder
- Prof. Gregory Seregin
- Dr. Neil Laws
- Prof. Stuart White
- Prof Neil Strickland (External Examiner)

Prof John Billingham (External Examiner)

• **Assessors:**

Prof. Konstantin Ardakov  
Dr Rafael Bailo  
Dr Immanuel Ben Porat  
Dr Joshua Bull  
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